SIMULTANEOUS EQUATIONS AND BOOLEAN ALGEBRA IN THE ANALYSIS OF JUDICIAL DECISIONS

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INTRODUCTION

In the study of the dependence of judicial decisions on relevant facts, traditional methods of analysis have encountered limitations in solving salient problems. In many areas of law, comprehensive sets of facts have been specified by courts as relevant and controlling for reaching decisions, with the understanding that some combinations of these facts would lead to decisions in favor of one party, whereas other combinations would lead to decisions in favor of the opposing party. Beyond the association of some combinations of these facts with decisions which already have been reached, it is not apparent, however, what decisions can be expected on the basis of other combinations of the specified facts. For example, in reviewing administrative decisions in workmen's compensation cases, appellate courts have indicated that the award or the denial of compensation must be decided on the basis of the particular combination of such facts as the nature of the injury, the conditions under which the accident or the harmful act occurred and became known, the health record of the claimant prior to the injury, and the evidence obtained from expert and lay testimony.

From a series of cases which already have been decided, it can be ascertained to what decisions some combinations of the relevant facts lead. But it cannot be readily inferred what decisions other combinations of these facts would justify. In order to appreciate that this problem is not a unique feature of any particular area of law, it is advisable to consider another example. In the involuntary confession cases arising under the due process clause of the fourteenth amendment, the Supreme Court of the United States has clearly stated that each decision depends on the facts pertaining to the pressure applied to the petitioner and to his inability to resist such pressure. Cases which already have been decided indicate what decisions have been reached on the basis of some combinations of such facts. They do not show, however, what decisions would correspond to other combinations of these facts.

The general problem in the analysis of all decisions which depend on various combinations of specified facts is, therefore, the following: to obtain a precise and exhaustive distinction between combinations of facts that lead to decisions in favor of one party and combinations of facts that lead to decisions in favor of the opposing party.

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party. This distinction can be obtained with the aid of mathematical models. To be sure, it can also be described verbally. But such a description would be extremely complex, and—in any case—it could be formulated only after a mathematical model has been designed.

For the purpose of obtaining the indicated distinction, two alternative models will be introduced in this presentation. One is based on simultaneous equations, and the other utilizes Boolean algebra. The models will be first discussed separately, and then their respective advantages and disadvantages will be compared. In conclusion, attention will be directed to the purposes, the limitations, and the implications of their use in the analysis of judicial decisions.

I

A Model Based on Simultaneous Equations

Before examining situations which actually are encountered in judicial decisions that depend on various combinations of specified facts, it will be helpful to consider a hypothetical example. Assume that in a given area of law facts $f_1$, $f_2$, and $f_3$ have been accepted by a court as relevant and controlling for the decision of cases. Assume also that these facts have different “weights” in determining the decision, and that these weights are represented by $x_1$, $x_2$, and $x_3$, respectively. Assume, furthermore, that three cases already have been decided. In Case 1, all three facts, i.e., $f_1$, $f_2$, and $f_3$, were present, and the decision was in favor of the party seeking redress. In Case 2, facts $f_1$ and $f_3$ were present, and the decision again was in favor of the aggrieved party. But in Case 3, facts $f_1$ and $f_2$ were present, and the decision was against the party seeking redress. What decision can be expected on that basis in a case in which facts $f_2$ and $f_3$ are present? Neither the stated rule that facts $f_1$, $f_2$, and $f_3$ shall be controlling for the decision, nor any of the decisions which already have been reached, offers an answer to this question. If it can be assumed, however, that the available decisions as well as future decisions form a consistent pattern of judicial action, a set of simultaneous equations can be written, and the solution of these equations provides an answer to this very question.

In the proposed set of simultaneous equations, each case is represented by one equation, and the decision of the case is treated as a function of the combination of controlling facts. Accordingly, the facts of the case are the independent variables in the equation, and the decision is the dependent variable. In this fashion, the cases which already have been decided form the desired set of simultaneous equations, in which the weights of the facts are the unknowns. As these equations are solved, a weight is found for each controlling fact, and—as these weights are substituted in the equation which represents a new case—a numerical value for the particular combination of facts and for the corresponding decision is obtained. Moreover, since the weights of the facts now are known, a numerical value for any combination of these facts and for its corresponding decision can be determined.
In order to illustrate this method in its most basic form, the following assumptions are now made with regard to the hypothetical example which has been introduced: (1) The combination of facts is linear, i.e., the relationship among the facts is additive. Further comments about this assumption will be made later. (2) The cases are decided by a court consisting of nine judges, and all judges participate in all decisions. This assumption also will receive further attention later. (3) Each decision is represented by the number of votes of judges favorable to the party seeking redress. The reason for this assumption is that the decisions of multi-judge courts do not necessarily constitute two opposite extremes, but are characterized by various degrees of support (or lack of support) of the aggrieved party by the votes of the judges. (4) Seven judges voted in favor of the party seeking redress in Case 1, five voted in favor of the aggrieved party in Case 2, but only three supported the party seeking redress in Case 3. The three decided cases in the hypothetical example then can be stated in terms of the following simultaneous equations:

\[ f_{11}x_1 + f_{12}x_2 + f_{13}x_3 = 7 \]
\[ f_{21}x_1 + f_{22}x_2 + f_{23}x_3 = 5 \]
\[ f_{31}x_1 + f_{32}x_2 + f_{33}x_3 = 3 \]

In this set of equations, only the weights of the facts, i.e., \( x_1 \), \( x_2 \), and \( x_3 \), are the unknowns. The coefficients \( f_{ij} \) (\( i = 1, 2, 3; j = 1, 2, 3 \)) are known. They indicate the presence or absence of the respective facts in the cases. More specifically, \( f_{ij} \) indicates whether fact \( j \) is present or absent in case \( i \). If the fact is present in the case, \( f_{ij} = 1 \); if it is absent \( f_{ij} = 0 \); if the fact has more than one manifestation in the case—for example, more than one accident occurred in a workmen’s compensation case—\( f_{ij} \) has a multiple value of 1. Accordingly, in Case 1 of the hypothetical example, \( f_{11} \), \( f_{12} \), and \( f_{13} \) each has a value of 1. In Case 2, \( f_{21} \) as well as \( f_{23} \) has a value of 1. But—since \( f_3 \) is absent in Case 2—\( f_{22} = 0 \), and this means that the second term in the second equation vanishes. In Case 3, \( f_{31} \) as well as \( f_{32} \) equals 1. However, \( f_{33} = 0 \) because \( f_3 \) is absent in Case 3, and this means that the third term in the third equation also vanishes. The simultaneous equations which represent the decided cases in the hypothetical example can be restated, therefore, in the following form:

\[ x_1 + x_2 + x_3 = 7 \]
\[ x_1 + x_3 = 5 \]
\[ x_1 + x_2 = 3 \]

These equations can be interpreted as follows: In Case 1, the presence and relative importance of facts \( f_1 \), \( f_2 \), and \( f_3 \) persuaded seven judges—i.e., a sufficient majority of the nine-judge court—that a decision in favor of the party seeking redress should be rendered. In Case 2, the presence and relative importance of facts \( f_1 \) and \( f_3 \) persuaded a smaller, but still sufficient, majority of five judges that a decision in favor of the aggrieved party was in order. But in Case 3, the presence and
relative importance of facts $f_1$ and $f_2$ gave the party seeking redress the support of only three judges, i.e., a decision in his favor was not reached.

As these equations are solved according to the basic rules of algebra, a weight of $x_1 = 1$ is obtained for fact $f_1$, a weight of $x_2 = 2$ for fact $f_2$, and a weight of $x_3 = 4$ for fact $f_3$. By substituting these weights in the equation which represents the new case in the hypothetical example, a numerical value for the combination of facts $f_2$ and $f_3$ and for the corresponding decision—i.e., six votes in favor of the party seeking redress—is obtained. Moreover, a numerical value for any other possible combination of the controlling facts (including any single fact) and for its corresponding decision can now be determined. In this fashion, a precise and exhaustive distinction between combinations of facts which call for a decision in favor of the aggrieved party and combinations of facts which require a decision against the party seeking redress is obtained.

In reality, however, judicial decisions depend on more extensive and more complex combinations of facts than are indicated in the foregoing illustration. For example, in the workmen's compensation cases which have been reviewed by the Connecticut Supreme Court of Errors, 19 relevant and controlling facts appear in the opinions of the court. In the involuntary confession cases decided by the United States Supreme Court, 22 relevant and controlling facts can be identified. It can be readily seen that in these areas of law simultaneous equations for 19 and 22 unknowns, respectively, must be solved in order to obtain the weights of the facts. Moreover, it is apparent from the basic rules of algebra that at least as many equations—i.e., cases which already have been decided—as unknowns must be available for that purpose. With a view to the situations that actually are encountered, the simultaneous equations then would have to be written in the following general form:

$$
\begin{align*}
 f_{11} x_1 + f_{12} x_2 + \ldots + f_{1j} x_j + \ldots + f_{1n} x_n &= D_1 \\
 f_{21} x_1 + f_{22} x_2 + \ldots + f_{2j} x_j + \ldots + f_{2n} x_n &= D_2 \\
 & \vdots \\
 f_{i1} x_1 + f_{i2} x_2 + \ldots + f_{ij} x_j + \ldots + f_{in} x_n &= D_i \\
 & \vdots \\
 f_{N1} x_1 + f_{N2} x_2 + \ldots + f_{Nj} x_j + \ldots + f_{Nn} x_n &= D_N.
\end{align*}
$$

In this form, the simultaneous equations apply to any number of facts and their weights ($j = 1, 2, \ldots, n$) in any number of cases ($i = 1, 2, \ldots, N$). As in the set of equations for the hypothetical example, the only unknowns are the weights of the facts, $x_j$. Each numerical value of $f_{ij}$ (0, 1, or multiple values of 1), which indicates the presence or absence of fact $j$ in case $i$, is known. Likewise, each numerical value of $D_i$, which indicates the decision in case $i$ in terms of the votes favorable to the party

For a detailed analysis of the Connecticut workmen's compensation cases, the involuntary confession cases, and the right to counsel cases in terms of the proposed method, see Kort, Content Analysis of Judicial Opinions and Rules of Law, in Glendon A. Schubert (Ed.), Judicial Decision-Making 133-97 (1963).
seeking redress, is known. In determining the value of $D_1$ for each case, certain
criteria for counting the votes of the judges must be observed. Only the votes of
judges who accept the facts as stated in the opinion of the court, and who support the
contentions of the party seeking redress on that ground, can be included in the
numerical value which represents the decision ($D_1$). If a judge writes a concurring
opinion in which he justifies his position in the case only on jurisdictional grounds,
his vote cannot be included; for his contribution to the decision is not a part of the
decision as a function of the applicable combination of facts. Furthermore, if a judge
supports the aggrieved party in a concurring opinion by accepting more facts than
are stated in the opinion of the court, his vote also cannot be included; for no
indication is given what his position would be on the facts which are accepted in
the opinion of the court, i.e., the facts which are controlling for the particular de-
cision. But, if a judge supports the party seeking redress by relying on fewer facts
than are stated in the opinion of the court, his vote can be included. This is
justifiable, because obviously the judge also would support the aggrieved party on
the basis of more facts in his favor.\footnote{These considerations have an important ramification for the meaning of a majority of votes in terms of this method of analysis. What constitutes a “majority” must be determined in relation to the number of participating judges in the sense in which “participation” just has been defined. In some situations, this may lead to a misclassification of decisions. Assume, for example, that—in a case before a nine-judge court—three judges support the decision in favor of the party seeking redress on the basis of the facts, two judges concur on jurisdictional grounds, and four judges oppose the decision on the merits of the case. In accordance with the criteria which have been advanced, this means that only three out of the seven votes of “participating” judges can be counted in favor of the aggrieved party. In this fashion, a decision against the party seeking redress is indicated, contrary to the actual decision. This limitation of the proposed method must be recognized. Empirically it has been found, however, that this limitation does not seriously impair the analysis of judicial decisions in terms of the proposed method. See Kort, supra note 1.}

A unique and perfect solution of the simultaneous equations can be obtained,
provided that the number of cases is equal to the number of facts—i.e., $N=n$, and
provided that the equations contain sufficient information. How the equations can
be solved if there are more cases than facts—i.e., $N>n$—will be discussed in a
moment. But what is of primary concern now is that—in most areas of law in
which the decisions depend on the combinations of facts—the available cases do not
provide sufficient information, even if their number is equal to or exceeds the
number of facts. In other words, the equations which represent the cases do not
contain sufficient information for obtaining a unique solution for the unknowns.
For the purpose of illustration, assume that Case 2 in the hypothetical example
consists of a combination of two manifestations of $f_1$ and $f_2$, and that six judges
vote in favor of the party seeking redress. With the substitution of the known values
of $f_{ij}$, the equations then would have to be written as follows:

\[
\begin{align*}
x_1 + x_2 + x_3 &= 7 \\
2x_1 + 2x_2 &= 6 \\
x_1 + x_2 &= 3.
\end{align*}
\]
The number of equations equals the number of unknowns, but a unique solution for \( x_1, x_2, \) and \( x_3 \) cannot be obtained. This is due to the fact that the second and third equations actually state the same relationship between \( f_1, f_2, \) and the corresponding decisions. Consequently, the three equations provide information on only two relationships in a situation that involves three facts. In the more complex combinations of facts that are encountered in actual situations, the lack of sufficient information is, of course, far less obvious than in the given hypothetical example. The problem of insufficient information for a unique solution of the equations can be overcome, however, by restating the original independent variables in the equations—i.e., the relevant and controlling facts in the cases—in terms of a new set of independent variables, which are called factors. The method by which the facts can be reduced to factors is known as factor analysis.

For the purpose of illustration, it will be helpful to consider two examples which already have been mentioned, namely, the workmen's compensation cases reviewed by the Connecticut Supreme Court of Errors and the involuntary confession cases decided by the Supreme Court of the United States. Some of the relevant and controlling facts which appear in the workmen's compensation cases can be described as follows: an accident or harmful act occurred in the course of an activity which was permitted by the employer; an accident or harmful act occurred in the course of an activity conducive to efficient work; an accident or harmful act occurred in the course of an activity which was indispensable for the performance of the work; an accident or harmful act occurred on the premises of employment, in an area annexed to the place of employment, or in an area where the work normally is performed; an accident or harmful act occurred during an activity which did not involve unnecessary, self-imposed hazardous conduct, such as taking a "joy ride" on a conveyor belt for unloading coal; the alleged injury became immediately apparent to the employee, as a result of an accident; the accident or the act which caused the alleged injury was observed by other persons; the alleged injury became immediately apparent to other observers, as a result of an accident or harmful act. To a large extent—although not exclusively—these facts can be restated in terms of a factor which can be called "a combination of facts which relate the alleged injury to an accident or observable harmful act."

Various methods of factor analysis have been developed. For the problem under discussion, Hotelling's Iterative Method of Factoring, also known as the Principal Components or the Principal Axes Method, is the most desirable method for locating the factors in terms of which the controlling facts can be restated. See Hotelling, Analysis of a Complex of Statistical Variables into Principal Components, 29 J. Educ. Psychology 417-44, 498-520 (1933); Hotelling, Simplified Calculation of Principal Components, 1 Psychometrika 27 (1936). For a complete exposition of this method, with regard to mathematical proof as well as application, see L. L. Thurstone, Multiple Factor Analysis 480-503 (1947). For restating the combination of circumstances in terms of factors in each case, the Shortened Estimation Method is most suitable. See Lederman, On a Shortened Method of Estimation of Mental Factors by Regression, 4 Psychometrika 199 (1939), and Harman, On the Rectilinear Prediction of Oblique Factors, 6 Psychometrika 29 (1941). For a complete exposition of this method and its relation to the Complete Estimation Method, with regard to mathematical proof as well as application, see Harry H. Harman, Modern Factor Analysis 338-56 (1960).
In the involuntary confession cases, some of the relevant and controlling facts which can be identified are the following: there was a delay in the formal presentation of charges; the defendant was detained incommunicado; the defendant was not advised of the right to remain silent; the defendant was not advised of the right to counsel; the defendant did not have any consultation with counsel prior to the challenged confession. These facts can be restated to a substantial degree in terms of a factor which can be described as "a tactic to keep the defendant in isolation, uninformed of the charges against him, and uninformed of his procedural rights." These examples provide an understanding of the intuitive meaning of restating facts in terms of factors. It should be noted, however, that the applicable factors actually are found by relying exclusively on the rigorous mathematical techniques which factor analysis employs. It also should be noted that—in addition to solving the problem of insufficient information in the original set of simultaneous equations—factor analysis fully explores the mutual dependence or independence of the facts.

On the basis of the restatement of the relevant and controlling facts in terms of factors, the decision in each case becomes a function of the combination of factors. Accordingly, the original set of equations can be restated in terms of a new set of equations, in which the factors in each case are the independent variables, and in which the decision is the dependent variable. In general terms, the new set of equations then can be written as follows:

\[
F_{11}X_1 + F_{12}X_2 + \ldots + F_{1j}X_j + \ldots + F_{1m}X_m = D_1 \\
F_{21}X_1 + F_{22}X_2 + \ldots + F_{2j}X_j + \ldots + F_{2m}X_m = D_2 \\
\vdots \\
F_{Ni}X_1 + F_{Ni2}X_2 + \ldots + F_{Ni}X_j + \ldots + F_{Nim}X_m = D_N.
\]

These equations apply to any number of factors and their weights (\(j = 1, 2, \ldots, m\)) in any number of cases (\(i = 1, 2, \ldots, N\)). Since the object of the factor analysis is to restate the independent variables in the original set of equations in terms of fewer independent variables in the new set of equations, it is expected that the number of factors, \(m\), is smaller than the number of facts, \(n\). Each numerical value of \(F_{ij}\), which is the factor estimate of factor \(j\) in case \(i\) (i.e., the measure of the degree to which factor \(j\) appears in case \(i\)), is known; it has been obtained as a result of the factor analysis. Likewise, each numerical value of \(D_i\), which indicates the decision in case \(i\) in terms of the votes of the judges favorable to the party seeking redress, is known. Consequently, the only unknowns in the new set of equations are the weights of the factors, \(X_j\). Since the facts have been restated in terms of factors, the number of the new equations, \(N\), exceeds the number of unknowns, \(m\).
But the equations can be solved by the method of least squares, which offers the best possible approximation to a perfect solution.

As new cases arise, the applicable facts can be reduced to the factors which have been identified, and—since the weights of the factors now are known—the decisions can be determined by substituting the weights of the factors in the equations which represent the new cases. It is in this respect that the proposed method for analyzing judicial decisions in the indicated areas of law provides a basis for prediction. Of more fundamental importance, however, is the criterion which this method offers for a precise and exhaustive distinction between combinations of facts that call for a decision in favor of the aggrieved party and combinations of facts that require a decision against the party seeking redress. For each combination of facts can be restated in terms of a combination of factors, and the weights of the factors and the degrees to which the factors represent the facts provide the numerical value for the decision.

As far as the application of the proposed method to the analysis of judicial decisions is concerned, it should be noted that the use of a high speed electronic computer is imperative. The solution of simultaneous equations with 20 or more unknowns, which must be expected in the analysis of decisions in the indicated areas of law, would be prohibitive without the aid of a computer. Moreover, the particular method of factor analysis which is recommended here involves such extensive iterative matrix multiplications that a computer becomes indispensable. It would be proper to say, therefore, that—from the viewpoint of all practical considerations—the proposed method could not be employed without reliance on a computer. It will be seen in a moment that these considerations also are pertinent for the analysis of judicial decisions in terms of Boolean algebra. In this respect, both models provide one of many examples of how research in law and the social sciences has been revolutionized by the invention of the electronic digital computer.

II

A MODEL BASED ON BOOLEAN ALGEBRA

The area of mathematics which is known as Boolean algebra was first developed by the British mathematician George Boole during the nineteenth century. Various references for the method of least squares are available. A convenient source, which contains a concise description and explanation of this method, is Gerhard Tintner, Mathematics and Statistics for Economists 273-86 (1953).

Although computers can design new methods of analysis, it should be carefully noted that—in this instance—the computer merely executes mathematical and statistical techniques which have been designed by human beings. That the computer performs in this process a task which practically cannot be achieved by human beings just has been seen. If, however, a statement like "decision prediction by computers" is used, the necessary qualifications of such a statement must be clearly understood. The view which was expressed in Wiener, Decision Prediction by Computers: Nonsense Cubed—and Worse, 48 A.B.A.J 1023 (1962) is based on fundamental misconceptions and presents a distorted description of the use of computers in legal research. Further comments about that article will be made in subsequent footnotes.

*George Boole, The Laws of Thought (1854).
can be employed in the form of compound statements, using symbolic logic, as well as in the context of the theory of sets. In both forms, Boolean algebra has been applied by Reed C. Lawlor to the analysis of judicial decisions that depend on various combinations of specified facts. It is on the basis of Lawlor's study that the model which uses Boolean algebra for analyzing these decisions now will be examined.

For the purpose of initial explanation, it will be convenient to refer again to the hypothetical example which was introduced earlier. It will be recalled that the cases in this example contain the following combinations of facts: \( f_1, f_2, \) and \( f_3 \) in Case 1; \( f_1 \) and \( f_3 \) in Case 2; and \( f_2 \) and \( f_3 \) in Case 3. It also will be recalled that the decisions in Case 1 and Case 2 are in favor of the party seeking redress, but that the decision in Case 3 is against the aggrieved party. It will be desirable now to change Case 1 to a case in which only \( f_2 \) and \( f_3 \) are present, but in which the decision is the same, and to include in the example another case, in which only fact 3 is present, and in which the decision is against the aggrieved party. An inspection of these four cases shows that the decision is in favor of the party seeking redress, \( D_{pro} \), if and only if \( f_3 \) and either \( f_1 \) or \( f_2 \) are present in the case. Accordingly, the following compound statement, using symbolic logic, can be written:

\[
D_{pro} = f_3 \land (f_1 \lor f_2).
\]

The symbols which are used in this statement have the following meanings:

\( \leftrightarrow \) represents the “biconditional” relationship “... if and only if ...”; \( \land \) denotes logical “conjunction,” i.e., “and”; \( \lor \) represents logical “disjunction,” i.e., “or,” but more specifically—it means “one or the other or both.” It can be seen, therefore, that this compound statement indicates that the decision is in favor of the aggrieved party if and only if fact 3, and either fact 1 or fact 2, or both fact 1 and fact 2 are present in the case. In this form, the statement provides a precise and exhaustive distinction between \( pro \) and \( con \) decisions (i.e., decisions in favor of and against the party seeking redress) in the hypothetical example. It should be noted that, like the method which uses simultaneous equations, the present method treats the decision as a function of controlling facts. Moreover, like the former method, the latter method can accommodate sets of consistent cases in which the number of cases exceeds the number of facts. But, unlike the method which is based on simultaneous equations, this method is concerned only with logical relationships, and does not assign numerical weights or values to combinations of facts and decisions.

The foregoing compound statement also can be written in a different form, using in part the concept of sets and subsets. It can be said that the facts in the hypothetical example form a “set,” which can be described by using the following notation:

\( \text{See J. G. Kemeny, J. L. Snell & G. L. Thompson, Finite Mathematics i, 69 (1957).} \)

$S = \{f_1, f_2, f_3\}$. This expression indicates that set $S$ contains the elements $f_1$, $f_2$, and $f_3$. Any combination of these elements forms a “subset.” Accordingly, $S_a = \{f_1, f_2\}$ would be the subset which contains elements $f_1$ and $f_2$. On this basis, the compound statement which provides the distinction between pro and con decisions in the hypothetical example also can be written as follows:

$$D_{pro} = f_3 \land L(1, S_a).$$

This statement has the following meaning: the decision is in favor of the party seeking redress if and only if fact 3 and at least (L) one of (,) the elements of subset $S_a$, consisting of fact 1 and fact 2, are present in the case. In this form, the statement offers the same precise and exhaustive distinction between pro and con decisions which it provided in its earlier form.

It must be noted again, of course, that in reality judicial decisions depend on more extensive combinations of facts than are suggested by the foregoing illustration. As applied to actual situations, the Boolean equation—i.e., the compound statement which provides the distinction between pro and con decisions—is therefore more complex. For example, in his study of the right to counsel cases decided by the Supreme Court under the Betts rule, Lawlor has used 39 facts. These facts are relevant and controlling for the decision whether or not a petitioner had been deprived of his right to counsel in a state criminal proceeding, in violation of the due process clause of the fourteenth amendment. Lawlor found that the following Boolean equation provided a distinction between the pro and con cases that had been decided:

$$D_{pro} = (f_{11} \lor f_{12}) \land f_{19} \land [L(1, S_a) \lor L(5, S_b)].$$

The terms in this expression represent certain facts and subsets of facts which apply to the right to counsel cases. Facts 11, 12, and 19 can be described, respectively, as follows: $f_{11}$—the petitioner had no assistance of counsel at the time of arraignment; $f_{12}$—the petitioner had no assistance of counsel between the time of arraignment and the trial, or between the time of arraignment and the plea of guilty; $f_{19}$—the petitioner never waived explicitly the right to counsel. Subset $S_a$ contains two facts: (1) the petitioner was convicted of a crime subject to capital punishment; (2) a jurisdictional issue or complicated charges were involved. Subset $S_b$ contains 32 facts, which pertain to the personal handicaps of the petitioner, to circumstances relating to the denial of representation by counsel, and to other procedural irregularities. On this basis, the Boolean equation states the following relationship: The decision in a right to counsel case is in favor of the petitioner if and only if fact 11 or fact 12 (or both), and fact 19, and at least one element in subset of facts $S_a$ or at least five elements in subset of facts $S_b$ (or both) are present in the case. Otherwise the decision is against the petitioner, $D_{con}$.

The form of the Boolean equation which distinguishes between \( pro \) and \( con \) decisions varies considerably in different areas of law. In some instances it might state a relationship exclusively in terms of conjunctions, whereas in other instances it might state a relationship exclusively in terms of disjunctions, or in terms of both conjunctions and disjunctions. In some instances it might contain a condition to the effect that all elements of a subset of facts must be present, whereas in other instances it might require the presence of only a given minimal number of elements in a subset. How can it be determined then which Boolean equation applies to a given area of law? Initially, it is not known which combination of facts constitutes the compound statement that distinguishes between \( pro \) and \( con \) decisions. If there are 20 relevant and controlling facts, the number of possible combinations of facts is \( 2^{20} \), i.e., approximately one million. If there are 39 relevant and controlling facts—as in Lawlor's study of the right to counsel cases—the number of possible combinations of facts is \( 2^{39} \), i.e., approximately 100,000,000,000. Even though—as will be seen in a moment—not every possible combination of facts has to be explored in order to obtain the applicable Boolean equation, the number of combinations which actually have to be examined still would make human inspection prohibitive. However, the systematic search for the Boolean equation can be performed by an electronic computer, and in this fashion the desired distinction between \( pro \) and \( con \) decisions is obtained. Computer programs which are designed to locate the applicable equation for any area of law in which the decisions are a function of facts have been developed independently by Lawlor and Kort.

An explanation has to be given now why not every possible combination of facts has to be examined in order to determine the Boolean equation which distinguishes \( pro \) and \( con \) decisions. The facts which are relevant and controlling for the decisions in the areas of law under consideration are \textit{monopolar}, or can be restated in monopolar form. In other words, all the facts favor the party seeking redress, or can be restated as facts favorable to the aggrieved party, and thus point in one direction. The facts which already have been mentioned as examples in the workmen's compensation cases, in the involuntary confession cases, and in the right to counsel cases are \textit{monopolar} in this sense. All these facts favor the claimant in a workmen's compensation case in obtaining an award, or favor the petitioner before the Supreme Court in obtaining a reversal of his conviction. A fact which is disadvantageous to the party seeking redress can be accommodated by including its opposite in all the cases in which it does not appear. For example, "previous experience in court proceedings" is a fact which does not favor the prospects of a petitioner to obtain a reversal of his conviction in a right to counsel case. Instead of identifying this fact in a case to which it applies, the fact "\textit{no} previous experience in court" can be included in the other cases, and the same relative standing of cases then is maintained. In this fashion, all the relevant and controlling facts can be stated in monopolar form.\(^\text{10}\)

\(^\text{10}\) With regard to the analysis of judicial decisions in terms of simultaneous equations, it should be
Since the facts are monopolar in the direction of favoring the party seeking redress, it can be said that a case $C_{i+a}$, which contains all the facts that are present in a case $C_i$ and other facts, has a higher rank ($Ra$) than case $C_i$. Moreover, it can be said that, if both cases contain the same facts, they have equal rank. On that basis, the following relationship can be stated:

$$[Ra(C_i) \leq Ra(C_{i+a})] \rightarrow (D_{pro}, i \rightarrow D_{pro}, i+a).$$

In this expression, the symbol "$\leq$" means that the term which follows the symbol is "higher than or equal to" the term which precedes the symbol. The notation "$\rightarrow$" represents the "conditional," and should be read as "if ... then ..." or "... implies ..." Accordingly, the following meaning can be given to the expression: If case $C_{i+a}$ has a rank higher than or equal to the rank of case $C_i$, then a pro decision in case $C_i$ implies a pro decision also in case $C_{i+a}$. This relationship assumes that the decisions constitute a consistent pattern of judicial action. In other words, it assumes that the decision in a case is as favorable to the party seeking redress as the decision in another case which is equally or less meritorious. But whether or not the decisions actually form a consistent pattern of judicial action can be initially determined by inspection, and this is a task which also can be assigned to a computer.

For con decisions, a similar relationship can be stated:

$$[Ra(C_i) \leq Ra(C_{i+a})] \rightarrow (D_{con}, i \rightarrow D_{con}, i).$$

This expression means that, if case $C_{i+a}$ has a rank higher than or equal to case $C_i$, then a con decision in case $C_{i+a}$ implies a con decision also in case $C_i$. Again it is assumed here that the cases constitute a consistent pattern of judicial action, and again it should be noted that the actual existence or nonexistence of such a pattern of consistency can be initially determined. It also should be understood that the subscripts $i$ and $i+a$ do not necessarily indicate the chronological sequence in which the cases are decided.

In order to see how these relationships apply to the location of the Boolean equation which provides the distinction between pro and con decisions, it is desirable to consider another hypothetical example. Assume that the decisions are a function of four relevant and controlling facts, $f_1, f_2, f_3$, and $f_4$. Sixteen different combinations of facts ($2^4$) and corresponding cases then are possible. Assume, furthermore, that 10 of the cases representing these combinations of facts already have been decided. In the list which follows, it is indicated which cases have been decided—and how they were decided—by using the notation pro and con. Moreover, set notation is used for representing the cases, in view of the fact that each case can be regarded as a subset of the total set of facts. The cases are not listed in the chronological order of the decisions.

noted that the facts are monopolar, but that the factors may be bipolar, i.e., they may point in two opposite directions. However, the analysis of the decisions in terms of Boolean algebra does not involve a restatement of facts in terms of factors.
In this example, the applicable Boolean equation can be easily found without the use of a computer. Nevertheless, the procedures which apply to the location of the equation in this simple example are the same as the procedures which have to be employed in finding the equation for combinations involving 20 to 40 facts, where the numbers of possible combinations range from one million to 100 billions. For this reason, a detailed examination of this example will be helpful.

It can be seen that case C7 in the example was decided in favor of the party seeking redress. Furthermore, it can be seen that cases C8 and C1, which also were decided in favor of the aggrieved party, have higher ranks than case C7, inasmuch as they contain all the facts that are present in C7 as well as other facts. If the cases form a consistent pattern of judicial action, the combinations of facts in C8 and C1 will not have to be considered as possibilities for the applicable Boolean equation, because any equation which designates C7 as a pro decision also designates C8 and C1 as pro decisions. And—as already has been indicated—whether or not the cases actually form a consistent pattern of judicial action can be initially determined by the computer. If the computer finds in the process of scanning the cases that there is a con case which has all the facts that are contained in a pro case, or all these facts and other facts, an inconsistency has been established. If such a case is not found, the pattern of decisions can be regarded as consistent. Moreover, if the pattern is consistent, the combination of facts in case C4 (which has not yet been decided) does not have to be considered, because any Boolean equation which designates C7 as a pro decision also designates C4 as a pro decision. Likewise, the combination of facts in C2 does not have to be considered, for any equation which designates C3 as a pro decision also designates C2 as a pro decision. This process of eliminating combinations of facts has been called by Lawlor "pro truncation." It should be recalled that this process assumes, of course, that the facts are monopolar.

Conversely, it can be shown how the process which Lawlor has characterized as "con truncation" applies to the example under consideration. Case C8 was decided against the party seeking redress. Cases C9, C10, and C11, which also were
decided against the aggrieved party, have lower ranks than case $C_8$, inasmuch as they do not contain all the facts that are present in $C_8$, and do not have any other facts. If the inspection by the computer shows that the cases form a consistent pattern of judicial action, the combinations of facts in $C_9$, $C_{10}$, and $C_{11}$ will not have to be considered as possibilities for the applicable Boolean equation, because any equation which designates $C_8$ as a con decision also designates $C_9$, $C_{10}$, and $C_{11}$ as con decisions. Moreover, if the pattern is consistent, the combinations of facts in $C_{10}$, $C_{14}$, $C_{16}$, and $C_{18}$ (which have not yet been decided) do not have to be considered, for any equation which designates $C_8$ as a con case also designates $C_{10}$, $C_{14}$, $C_{16}$, and $C_{18}$ as con decisions. Again it should be noted that the facts are assumed to be monopolar.

After applying pro truncation to the example under discussion, the Boolean equation which distinguishes between pro and con decisions then can be obtained by taking the "logical sum"\(^\text{11}\) of the pro cases $C_8$, $C_9$, and $C_7$:

$$D_{\text{pro}} \rightarrow (f_1 \land f_2) \lor (f_1 \land f_3) \lor (f_1 \land f_4).$$

This expression can be simplified to

$$D_{\text{pro}} \rightarrow f_1 \land (f_2 \lor f_3 \lor f_4).$$

It states that the decision is in favor of the party seeking redress if and only if fact 1 and, in addition, either fact 2 or fact 3 or fact 4 (or two or all of the latter three facts) are present in the case.

The Boolean equation also can be obtained by taking the logical sum of the con cases $C_8$ and $C_{12}$:

$$D_{\text{con}} \rightarrow \neg f_1 \lor (\neg f_2 \land \neg f_3 \land \neg f_4).$$

The symbol \(\neg\) in this expression is the "negation" of the term which it precedes; e.g., \(\neg f_1\) means not \(f_1\). The statement then means that the decision is against the party seeking redress if and only if fact 1 is absent in the case, or if none of facts 2, 3, and 4 are present. This statement has the same meaning as the statement in terms of the pro decisions.

Aside from providing a precise and exhaustive distinction between the pro and con decisions in this example, the Boolean equation also makes it possible to predict the decisions which have not yet been reached. Since the equation designates $C_2$ and $C_4$ as pro decisions, it can be predicted that these cases will be decided in favor of the party seeking redress, provided that future decisions will remain a part of the consistent pattern of judicial action which has been identified. With the same qualification, it can be predicted that cases $C_{10}$, $C_{14}$, $C_{16}$, and $C_{18}$ will be decided against the aggrieved party, inasmuch as the applicable Boolean equation designates $C_{10}$, $C_{14}$, $C_{16}$, and $C_{18}$ as con decisions. If—in actual situations—the cases which

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\(^{11}\) The term "logical sum" becomes fully plausible if it is noted that an alternative notation for disjunction is "\(\lor\)". But, in this sense "\(\lor\)" has the same meaning as "\(\lor\)", and is not equivalent to "\(\lor\)" in arithmetic.
have been decided do not provide enough information for an exhaustive distinction, the best possible approximations must be made.

The location of a satisfactory Boolean equation also involves a task which is more complex than taking the logical sum of the pro decisions and the logical sum of the con decisions. For it is necessary to determine the subsets of facts and the minimal numbers of their elements which satisfy the equation. In the example under consideration, it is relatively unimportant whether the applicable Boolean equation is stated as

$$D_{pro} \leftarrow f_1 \land (f_2 \lor f_3 \lor f_4)$$

or as

$$D_{pro} \leftarrow f_3 \land L(1, S_a).$$

where

$$S_a = \{f_2, f_3, f_4\}.$$

But where large subsets of facts are involved, the difference between stating the equation in a form which uses subsets and writing the equation in a form which uses exclusively individual facts is crucial. Consider, for example, Lawlor's Boolean equation for the right to counsel cases. Subset $S_b$ in this equation consists of 32 facts. If the last term in this equation would have to be written in a form using only individual facts, it would contain 201,376 disjunctions. It can be seen that such a statement would not be feasible. For practical purposes, it is therefore necessary to instruct the computer to identify the applicable subsets of facts and the minimal number of their elements which satisfy the Boolean equation.

In addition to pro and con truncation, a reduction in the number of combinations of facts which have to be explored in finding the Boolean equation is achieved by taking into account the cumulative effect of some facts. Consider, for example, the following two facts in the workmen's compensation cases: (1) an accident or harmful act occurred in the course of an activity conducive to efficient work; (2) an accident or harmful act occurred in the course of an activity which was indispensable for the performance of the work. If the first fact is represented by $f_i$ and the second fact by $f_{i+1}$, it can be said that $f_{i+1} \rightarrow f_i$, although $\sim (f_i \rightarrow f_{i+1})$. In other words, the occurrence of an accident in the course of an activity which is indispensable for the performance of the work implies that the accident also occurred in the course of an activity which was conducive to efficient work. On the other hand, the fact that the accident occurred in the course of an activity conducive to efficient work does not imply that it occurred in the course of an activity indispensable for the performance of the work, even though—according to the "law of contraposition"—$\sim f_i \rightarrow \sim f_{i+1}$. For this reason, the two facts have to be identified separately. But assume now that the search for the applicable Boolean equation considers the possibility that $f_{i+1}$ and some other facts must be present in a case for a pro decision.
In the process of this search, the combination \( \ldots f_j \land f_{j+1} \ldots \) does not have to be examined, because in any case in which \( f_{j+1} \) is present \( f_j \) also is present, and in any case in which \( f_j \) is absent \( f_{j+1} \) also is absent. In this fashion, a further reduction of the number of combinations of facts which have to be considered in locating the Boolean equation is obtained.

### III

#### A Comparison of the Two Models

Each of the models which has been introduced for the analysis of judicial decisions offers some advantages and some disadvantages in comparison with the other model. One significant advantage of the method based on Boolean algebra is that it does not have to make any assumption with regard to the linearity or non-linearity of the combination of facts. In the method based on simultaneous equations, this is an important consideration. For example, in the workmen's compensation cases the question arises whether a previous illness is a fact which adds to the likelihood of an occupational disease as indicated by the other facts, or modifies such likelihood in some other way. If the former is true, the relationship between the fact "previous illness" and the other facts would be additive, i.e., the combination would be linear. If the latter is true, the relationship between the fact "previous illness" and the other facts would not be additive, i.e., the combination would be non-linear.

Taking the possible combinations of many different facts into account, it is difficult to determine which are linear and which are non-linear. To be sure, the proposed method of analysis in terms of simultaneous equation actually does not make any assumptions regarding the linearity or non-linearity of the combinations of facts, inasmuch as the facts are restated in terms of factors. It does assume, however, that the combinations of factors in the cases are linear, and this is an assumption which conceivably could be refuted. On the other hand, the method of analysis in terms of Boolean equations makes no assumption in this respect, for it is concerned only with the simultaneous appearance or non-appearance of facts in the cases.

Another advantage of the method based on Boolean algebra is that it does not use numerical weights. It is true, of course, that the factor estimates and the weights of the factors in the method based on simultaneous equations are not determined arbitrarily, but are obtained by rigorous mathematical techniques. Moreover, it is plausible to assign numerical values to the decisions in terms of the votes of the participating judges. Nevertheless, the assumption is made that the votes of the judges are the best numerical index for the decisions, and the weights of the factors are obtained from equations in which this index is the dependent variable. On the other hand, the method which employs Boolean algebra does not make such an assumption. And this method does not ignore the degree of support of the aggrieved party by the different judges, inasmuch as it can be applied to the position of each
judge. Lawlor has demonstrated this aspect of the analysis in his experimental tests in the right-to-counsel cases.12

One of the main advantages of the method using simultaneous equations is that it provides the distinction between pro and con cases by obtaining a unique solution of the equations in which the facts have been restated in terms of factors. To be sure, the fact that a trial-and-error procedure has to be used to locate the applicable equation in the method based on Boolean algebra is not a disadvantage. For it has been shown that this elaborate task can be performed by a computer. But several criteria for the designation of subsets of facts in the Boolean equation that provides the desired distinction are available, and the question then arises on what grounds one criterion should be preferred to another. On the other hand, this problem does not arise in the method using simultaneous equations, for the solution which is obtained is the best possible solution in the least square sense.

A further advantage of the method based on simultaneous equations is that it fully explores the mutual dependence or independence of the facts by means of factor analysis. It is quite true that—as has been shown—the dependence of some facts on other facts can be explored in the form of logical implications. Nevertheless, the latter do not indicate the various degrees of mutual dependence which are obtained through the correlation measures in the factor analysis.

IV

THE PURPOSES, LIMITATIONS, AND IMPLICATIONS OF THE ANALYSIS OF JUDICIAL DECISIONS IN TERMS OF THE PROPOSED MODELS

In spite of the respective advantages and disadvantages of the two methods, identical purposes can be attributed to them. As has already been indicated, their main purpose is to provide a precise and exhaustive distinction between decisions which depend on combinations of facts that have been specified by the courts. It is in this fashion that the proposed methods of analysis offer information about the content and the application of rules of law which the verbal statements of these rules do not provide. For in deciding cases in the indicated areas of law, courts actually employ rules of law which state that the decisions shall be made on the basis of the combinations of facts that appear in the particular cases. The verbal statements of these rules specify which facts shall be regarded as relevant and controlling. But they do not specify which particular combinations of these facts call for a decision in favor of the party seeking redress, and which combinations of facts demand a decision against the aggrieved party. It is the absence of this information which has been criticized,13 and it is precisely this information which judges and lawyers need for appraising cases in terms of these rules.

13 For such criticism regarding the right to counsel cases, see Note, 33 Va. L. Rev. 731 (1947);
To be sure, this information could be provided by a verbal *restatement* of the applicable rule of law. However, such a restatement would be extremely complex and—from a practical point of view—extremely cumbersome. The verbal restatement would have to contain all the possible combinations of the controlling facts and the corresponding decisions, and it already has been seen that in actual situations the number of these combinations would range from one million to several billions. It is hardly conceivable that an intelligent human being could obtain a precise and exhaustive formulation of the applicable rule of law from this number of combinations without reliance on mathematical techniques and without the aid of a computer. It can be seen, therefore, that—even if a verbal restatement of the rule should be preferred—an analysis in terms of the proposed methods would be necessary before such a restatement could be formulated.

On the basis of the precise and exhaustive distinction between *pro* and *con* decisions, another main purpose of the proposed methods of analysis becomes apparent, namely, the prediction of new decisions. It has been shown how both methods can serve this purpose. Of course, prediction is possible only if it can be assumed that the consistent pattern of judicial action which has been detected in past cases will continue in the future. This points to an important limitation of the analysis of judicial decisions in terms of the proposed methods—a limitation which will receive further attention in a moment. Moreover, it must be clearly understood that the prediction of new decisions is *conditional* and not *unconditional*. In other words, the decisions are predicted on the assumption that the combinations of controlling facts which will be accepted by the court are known.

To predict which facts actually will be considered by the court in a given case is an entirely different task. If such a predictive device can be found, its combination with the present methods would permit an *unconditional* prediction of decisions. But even conditional prediction offers at least two advantages: (1) It enables an attorney to anticipate rationally—and not merely intuitively—which combinations of controlling facts would lead to a decision in favor of his client and which combinations would be insufficient for such a decision. On that basis, he could appraise more accurately the prospects of his client, and also would be in a better position to know which facts he should emphasize in his presentation to the court. (2) Conditional prediction makes it possible to determine whether or not a new decision represents a continuation of a pattern of judicial action which has been established in previous decisions. For after a new decision, the combination of facts that has been accepted by the court is, of course, known. And, certainly, the initial task of both methods of analysis is to explore whether or not a consistent pattern can be identified.

The proposed methods require not only an explanation of their purposes, but

Comment, 22 So. Cal. L. Rev. 259 (1949); Green, The Bill of Rights, the Fourteenth Amendment, and the Supreme Court, 46 Mich. L. Rev. 869, 898 (1948), and the reference to these sources in William M. Beany, The Right to Counsel in American Courts 194 (1955).
also a clear recognition of their limitations. One limitation just has been discussed, namely, that—at their present stage—the methods in question do not offer unconditional, but only conditional prediction. Another limitation is that the test of consistency and conditional prediction cannot be applied to a case in which a fact not previously encountered appears. However, if such a case is included in the analysis, subsequent cases in which this new fact is present can be tested for consistency and can be conditionally predicted. A further important limitation is that the proposed methods are not designed to predict doctrinal changes and the adoption of new rules of law. For example, these methods could not have predicted the overruling of Betts v. Brady—the case in which the Supreme Court had stated the rule that the decisions of the state right to counsel cases depend on the combinations of certain relevant and controlling facts.

If the utility of the proposed methods is questioned under these circumstances, the following considerations should be noted. First of all, the fact that the applicability of the methods to a given area of law is terminated by a doctrinal change does not affect in any way their applicability to other areas of law, where such doctrinal changes have not occurred. For example, the overruling of Betts v. Brady obviously has no consequences for the analysis of the involuntary confession cases and the workmen’s compensation cases in terms of the proposed methods. Secondly, even in the area of law in which the doctrinal change has occurred, an analysis in terms of these methods will have provided insights which otherwise could not have been obtained. As far as the right to counsel cases are concerned, this means that—
even though *Betts v. Brady* has been overruled—the analysis has demonstrated that a rule of law which has been condemned as a "nebulous standard," as a complex "ex post facto standard," and as an "arbitrary and capricious rule," actually has been employed with remarkable consistency. In other words, the proposed methods have revealed a pattern of judicial action which traditional methods of interpretation have not been able to detect.

It is the latter consideration which points to important implications of the proposed methods. In examining past decisions by means of these methods, no assumption is made regarding the existence or nonexistence of a consistent pattern of judicial action. Whether or not consistency does exist in a given area of adjudication is determined by the very use of the methods. And that they can detect the existence or nonexistence of consistency more accurately than traditional methods just has been seen. If a consistent pattern cannot be identified, it must be concluded that judicial action in the given area of law cannot be understood in terms of the dependence of decisions on various combinations of specified facts. A different interpretation of the decisions and an examination of other models for their analysis then would be appropriate. If, on the other hand, the decisions reveal a consistent pattern, the proposed methods make it possible to determine whether the consistency of judicial action in the given area of law can be explained in terms of *stare decisis,* or must be understood in terms of a pattern of regularity beyond the traditional meaning of this concept.

In an area of law in which a sufficiently large number of decisions have been rendered, the available cases can be chronologically divided into two halves, and an attempt then can be made to predict one-half of the cases from the other half. If the results show that it is possible to predict only the chronologically second half of the cases from the chronologically first half, the conclusion can be reached that *stare decisis*—in the sense of basing later decisions on earlier precedents—has been followed. If, on the other hand, the results indicate that the second half of cases can be predicted from the first half, and the first half can be predicted from the second half, the consistency of judicial action cannot be explained in terms of *stare decisis.* For consistency which can be detected in earlier decisions by prediction from later decisions obviously cannot be attributed to a process of basing earlier decisions on later precedents.

It seems, therefore, that consistency of judicial action—which in many instances

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18 See *supra* note 13.

19 On that basis, it can be seen that Wiener's statement, *supra* note 5, at 1024, to the effect that the first assumption in the proposed methods of analysis is that "the tribunal in question adheres to the doctrine of *stare decisis*" is false. Any argument along these lines would be valid only if it could be shown that *stare decisis* never is followed in the areas of law to which the proposed methods apply (not in the unrelated areas of law to which Mr. Wiener is referring). Apparently Mr. Wiener is not prepared to take such a position, for he warns cogently (*supra* note 5, at 1026) against any oversight in the search for precedent. Aside from these considerations, it will be seen in a moment that consistency in judicial action is not necessarily identical with *stare decisis* in the traditional sense.

20 This has been the case, for example, in the analysis of the Connecticut workmen's compensation cases. See Kort, *supra* note 1.
would appear to be an application of stare decisis—actually would have to be explained in terms of a pattern of regularity which differs from adherence to precedent. Such consistency would have to be understood in terms of an independent—although convergent—recognition and acceptance of similar standards of justice by different judges at different times. It is in this respect that the proposed methods of analysis provide new insights into a principle which has been a pillar of the common law.