ABSTRACT

Using the language of mathematics, Professor Polk Wagner has recently argued that the impossibility of fully appropriating the value of information in a rightsholder leads to the surprising conclusion that expanding the degree of control of intellectual property rights will, in the long run, increase the sum total of information not subject to ownership claims and therefore available as part of the cultural and technological base on which new growth and development can occur. Indeed, he claims that open information will grow according to the formula for compound interest, where the interest rate is 100% plus or minus a factor z supposedly related to creation incentives. This article demonstrates that Professor Wagner’s mathematical analysis is simply wrong and does not lead to any of the conclusions he reaches concerning the growth of open information. It also shows both the difficulties and the dangers of the lay use of the language of mathematics in resolving complex social problems even if one does the math correctly.

INTRODUCTION

Professor Polk Wagner has argued that the impossibility of fully appropriating the value of information in an intellectual property rightsholder, such as the owner of a patent or copyright, leads to the surprising conclusion that expanding the degree of control of intellectual property rights will, in the long run, increase the total information not subject to ownership claims. This “open information” is then available as part of the cultural and technological base on which new growth and development can occur. Professor Wagner’s argument is not that intellectual property rights are for limited times, so increased incentives generate more works that eventually enter the public domain, thereby

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1 Jack E. Brown Professor of Law, Sandra Day O’Connor College of Law, Arizona State University. The author is indebted to Professors Vincent Chiappetta and Aaron Fellmeth for many helpful comments on an earlier draft of this article.

increasing the supply of open information. Rather, Professor Wagner argues that the creation of any new work results in new open information indirectly associated with the new protected work but sufficiently removed from it that the new open information cannot be appropriated as part of the original intellectual property right. He claims that by giving more control over direct uses of the work to the rightsholder, we increase incentives for the creation of such works and thereby get more of them. Even though fewer direct uses can be made of protected works after the increase in control, the increase in the indirect supply of information will, in the long run, more than make up the difference. Thus, granting more control to authors and inventors and thereby increasing their incentives to create will result in more growth of open information than would occur under the status quo or under a system of reduced rightsholder control.

Professor Wagner’s paper contains much thoughtful and well reasoned analysis, but it suffers from a fatal flaw. He simply assumes that the formula for compound interest applies to the growth of open information, while his presentation, purporting to use the language of mathematics, seriously obfuscates his calculations. That is, he asserts that the amount of open information $O_n$ at the end of $n$ periods of time is equal to the amount that existed at some time in the past $O_1$ multiplied by $(1+z)^n$, where $z$ is a growth factor supposedly related to creation incentives. This mathematical formula accurately predicts the value of a bank account paying compound interest or the growth of bacteria in a medium of unlimited necessary nutrients. Does it, however, apply to the growth of open information—does information beget information like bank accounts beget money or bacteria beget offspring, and if so, how?

Professor Wagner does not attempt to justify his application of the formula for compound interest to information growth. This “straightforward” argument would be neither new nor, in itself, convincing, because it does not consider the loss of derivative works that are not created as a result of the extra control the new regime recognizes for rightsholders. Professor Wagner calls this Type III information; Type I is the copyright-protected work or patented invention itself, while Type II information is directly derived from Type I information as cumulative innovation, such as invention improvements or copyright derivative works. Wagner, supra note 2, at 1003–05.

There are, indeed, some serious studies attempting to show that knowledge has been growing exponentially. For example, Ray Kurzweil argues that the history of technology shows exponential growth in technological change; indeed, the growth is doubly exponential, in that the exponential growth factor itself has been growing with time. Ray Kurzweil, The Law of Accelerating Returns, KurzweilAI.net, Mar. 7, 2001.
demonstrates that the mathematics purporting to underlie Professor Wagner’s argument is simply wrong. It argues that application of the compound interest formula is absurd, especially pursuant to an incentives-based theory that looks, as his theory purports to do, to people—authors and inventors—for the creation of cultural and technological works. This article thus has two goals: First, it explains why Professor Wagner’s erroneous claim, which appears in one of our most prestigious law journals, must be rejected in the ongoing debate on the appropriate type and scope of intellectual property rights. Second, using Professor Wagner’s basic approach as an example, this article also shows both the difficulties and the dangers of the lay use of the language of mathematics to address complex social problems even if one does the math correctly.

I. THE WAGNER THEORY

¶4 Authors and inventors create new information. Our whole theory of intellectual property rests on the assumption that, by giving authors and inventors legal protection for information they create, we get more information of the types society deems desirable than we would have under a regime that allowed free copying of all generally available information. Intellectual property laws thus give information creators a degree of control over the information they produce as an incentive to produce it. But does information itself beget new information? That is, does the availability of information to human beings in itself lead to the creation of new information?

¶5 At first, the question seems absurd. Notwithstanding the human tendency to anthropomorphize, information by itself is inanimate and incapable of reproducing itself, let alone creating new information. Still, the current supply of information does, in a sense, lead to new information. Movies and books lead to movie and book reviews. Scientific papers lead to new research and new papers. Yesterday’s ball game leads to a new dynamic for today’s game. Such examples are literally without limit. While they all involve human intermediaries in the creation of the new information, some of the new information clearly would either be different or would not exist at all but for the earlier information. Therefore, the amount and quality of information we as a society generate at any given time must somehow depend on the size and quality of the existing information base. The amount of information available at any given time is thus a complex function of the incentives society gives to information producers and the quantity and quality of information on which those producers can build.

[http://www.kurzweilai.net/articles/art0134.html?printable=1](http://www.kurzweilai.net/articles/art0134.html?printable=1). A similar conclusion can perhaps be inferred from some of Isaac Asimov’s work. See infra note 27.
A. Professor Wagner’s Model

Professor Wagner models this phenomenon by considering a series of time periods of unspecified length during which creative authors or inventors produce information. The total information produced in such a time frame consists of: $i_I$, the Type I information relating to the core creations (e.g., patented inventions or copyright-protected works); $i_{II}$, the Type II information directly related to and derived from those core creations; and $i_{III}$, the Type III information that results from Types I and II information but is beyond the control of any intellectual property rights. Therefore, the total information produced during the given time period is $I = i_I + i_{II} + i_{III}$. Only the open information $O$ however, is freely available for later authors and inventors to use in new creations. All Type III information is by definition open, and certain percentages of Types I and II information are also open because intellectual property rights grant less than full control to the rightsholders. If the respective percentages of these two types of information that are open are $c_I$ and $c_{II}$, the amount of open information produced during this time period is

$$O = c_I i_I + c_{II} i_{II} + i_{III} \quad (1)$$

Equation (1) represents the status quo—the amount of new open information that will be created within the given time period if we make no changes in the incentive schemes of intellectual property law. Professor Wagner now multiplies Equation (1) by a factor $z$ reflecting the change in

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7 Because Professor Wagner does not expressly tell his readers that he applies the formula for compound interest to the growth of open information, it is necessary to present his purported mathematical analysis in some detail to uncover the absence of any basis, at least within his analysis, for doing so. While I find his mathematical notation clumsy, I adopt it here to allow easier comparison of my analysis to his. To keep the discussion as simple as possible, I also grant him most of his assumptions, challenging them only in footnote.

8 Type I information is the copyright-protected work of authorship or the patented invention itself. Type II information derives directly from such underlying creations, such as improvements in the patent context and derivative works, like movies made from books, in the copyright context. Type III information derives from the underlying creation but is beyond the control of the intellectual property rightsholder. The new technological avenues that are opened by pioneering inventions like the steam engine and the transistor or new genres of television shows that follow a successful first example are Type III information. Wagner, supra note 2, at 1003–06.

9 Some information may be open because rightsholders choose not to assert some of their rights, as with open-source software. Professor Wagner has not included this type of information in his model, and I ignore it as well. I can see no reason how including such information in the model would change any of the fundamentals.
creation incentives that results from increasing or decreasing rightsholder control:

\[ O(z) = z[c_1i_1 + c_{11}i_{11} + i_{111}] \] (2)

\(^8\) The incentives factor \( z \) is unity for the status quo; it goes up with increases in rightsholder control (increased incentives) and down with decreases in such control. Because \( i_1 \) is itself a function of \( z \), it seems like double counting to multiply again by \( z \) in Equation (2). Professor Wagner apparently assumes that \( i_1 \) is a constant and that all of the increase comes from linear multiplication by \( z \). Much of the rest of this article questions the apparent assumption of such a linear relationship.

\(^9\) Of course, the coefficients \( c_1 \) and \( c_{11} \) also vary with \( z \). They vary inversely with the change in control, that is, with \( z \), because by definition they represent the amounts of Type I and II information not, under intellectual property law, subject to the control of the rightsholder. For example, if the law changes to reduce the number of uses of copyright-protected works deemed “fair use,” a copyright owner will have more control over the uses that can be made of Type I information, so a smaller percentage of \( i_1 \) will be open information (\( c_1 \) decreases). On the other hand, to the extent greater copyright owner control increases the incentive to create new works, the amount of Type I information \( i_1 \) should increase. Similarly, if the notion of “expression” is expanded so that more works are considered infringing derivative works, more Type II information will be subject to copyright owners’ control, causing less of \( i_{11} \) to be open (\( c_{11} \) decreases).\(^{10}\) Presumably, less Type II information will also be created with the increase in rightsholder control because more such works will require a rightsholder’s permission and the increased transaction costs will prevent some works from ever being created.\(^{11}\) Note that nothing in Equation (2) implies a “feedback” of the amount of open information onto itself; that is, Equation (2) includes no term relating the amount of open information \( O \)

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\(^{10}\) Professor Wagner recognizes, but does not address in detail, the possibility that Type II information may also be, or become, Type I information. \textit{Id.} at 1018 n.97. For example, a movie made from a novel has much original material that is protected, if made with permission, as a derivative work. Presumably, the original elements of a protected derivative work constitute Type I information, while the elements that would, but for the copyright owner’s authorization, have made it an infringing derivative work are Type II information. Again, the critique of Professor Wagner’s theory contained herein does not depend on these details and therefore grants him, for present purposes, whatever assumptions he is making in this regard.

\(^{11}\) I address later some of the difficulties this Type I, II, III taxonomy of information has for a correct analysis of the open-information growth problem. It is considerably more complex than Professor Wagner’s model, even correctly implemented, can account for.
created during the period to the amount existing at the beginning of the period. It simply states that, if we increase control, and thereby creation incentives, we will increase society’s total amount of open information by the amount \( O \) calculated according to the equation.

**B. The Three Scenarios Considered**

Professor Wagner now considers three scenarios. Scenario 1 is the status quo, where he takes \( c_I \) to be 10% and \( c_{II} \) to be 40%; scenario 2 reduces rightsholder control, thereby opening up more Types I and Type II information to free use, and takes \( c_I \) to be 40% and \( c_{II} \) to be 60%; and scenario 3 gives perfect control over Types I and II information to the rightsholders, so that both \( c_I \) and \( c_{II} \) are zero. Apparently, he also takes his change-in-incentives factor \( z \) to be 1 for scenario 1 (the status quo), 2/3 for scenario 2 (reduction of incentives by 1/3) and 4/3 for scenario 3 (increase of incentives by 1/3).  

He then seeks to compare the increases in the

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12 Professor Wagner is not entirely clear about what he means by an increase in incentives (via increased creator control over the product). Because he sees an increase or decrease by 1/3 in the total information output \( I \) when \( z \) is increased or decreased, it appears that his definition of incentives is based on the results of whatever change in control he hypothesizes. That is to say, if we make a change in control that results in \( i_I \) output increasing by 1/3, then by definition, we have increased incentives, and therefore \( z \), by 1/3. There is no obvious reason why the coefficients in his Equation (2), \( c_I \) and \( c_{II} \), should change by the same or even a closely related percentage. Thus, when \( z \) decreases by 1/3 in Professor Wagner’s scenario 2, his \( c_I \) goes from 10% to 40%, which means a decrease in rightsholder control by 1/3 (from 90% to 60%). His \( c_{II} \) goes from 40% to 60%, which also means a decrease in rightsholder control by 1/3 (from 60% to 40%). I can think of no reason, and Professor Wagner supplies none, that a change in incentives measured by changes in output would bear any linear relation, let alone equality, to the changes in the level of control society gives over information Types I and II. Indeed, in Professor Wagner’s full control scenario 3, \( c_I \) and \( c_{II} \) go from 10% and 40%, respectively, to zero, representing an increase in rightsholder control of about 11% for Type I information (90% to 100%) and 67% for Type II information (60% to 100%). These changes for scenario 3 average out to a little more than a 1/3 increase in rightsholder control (39%—recall that he that he takes \( i_I \) and \( i_{II} \) to be equal), so he cannot be assuming a linear relationship, either. This detail would demand closer discussion if Professor Wagner’s basic theory represented the real world in any meaningful way. Indeed, he emphasizes that a crucial implication of his theory is the need for a closer examination of the relationship between control and incentives. See Wagner, supra note 2, at 1023. This critique is aimed at his assumption of exponential growth based on the incentive factor \( z \), so it matters little here exactly what \( z \) is or how it is derived.
amount of open information created during the given time period, assuming that \(i_1 = i_{II} = i_{III} = 1\).

11 Applying Equation (2), the increase in the amount of open information that results during time period 1 for each of the three scenarios is \(O_{11} = 1[0.1 + 0.4 + 1] = 1.5\) for scenario 1, \(O_{12} = 2/3[0.4 + 0.6 + 1] = 2/3 \times 2 = 4/3 = 1.333\) for scenario 2, and \(O_{13} = 4/3[0.0 + 0.0 + 1] = 1.333\) for scenario 3. How much open information is then produced in the second time period? By hypothesis, incentives for producers of works (by giving them the same, less, or more control over their creations) have not changed from the first time period. Under scenario 3 (increased incentives) the change has resulted in higher productivity by existing creators or in inducing more people to become creators. The opposite has occurred under the reduced incentives of scenario 2, and nothing has changed with respect to incentives for status quo scenario 1. Unless the total amount of open information itself somehow begets new information, perhaps by allowing the group of creators that now exists to work more efficiently in response to the change in incentives, we should get identical increases in open information in time period 2 and, indeed, in all successive time periods. Recall that Equation (2) does not incorporate any term representing efficiency “feedback” to creators based on the total amount of open information available to them to work with that increases their productivity.

13 This assumption would also require much more careful analysis if the underlying theory were otherwise valid. For present purposes, however, we may accept it, because it reflects only the initial conditions for the analysis. If we assume information really grows exponentially at a rate dependent on intellectual property incentives, and if we agree with Professor Wagner to look only to the long term, the initial conditions will make little difference, as long as we define “long” to be a period of sufficient length that the initial conditions are largely irrelevant. This is always possible with exponential growth. See infra note 19 and accompanying text.

14 Professor Wagner seems not to apply his incentive factor \(z\) to this first iterative period, so the numbers he gets for the increase in the amount in his first period are 1.5, 2, and 1 for the three scenarios, respectively. Wagner, supra note 2, at 1020–21. By hypothesis he has changed the incentives for scenarios 2 and 3. He has stated that the change in incentives will be reflected in changes in the coefficients \(c_I\) and \(c_{II}\), the amounts of \(i_I\) and \(i_{II}\), respectively, that cannot be appropriated by the rightsholder. But he asserts that the total number of works produced will also change, which is why he introduces the factor \(z\). Given that scenarios 2 and 3 both reflect a change in incentives, it would seem that the total amount of open information produced during the first period should take into account the assumed change in the number of works produced during the period, which means multiplying by \(z\). Fortunately, whatever choice is made on this issue does not affect any subsequent conclusions under either Professor Wagner’s method of analysis or the one presented here (because his assumption of exponential growth swamps all these details).
Assuming for the moment, therefore, that there is no such feedback (Professor Wagner’s theory does not postulate any), it is easy to see that open information under Equation (2) grows faster under the status quo and equally slower in both the reduced and the increased incentives scenarios (increasing by 1.5 in each time period for scenario 1 compared to 1.333 in the others).

C. The Leap to the Compound Interest Formula

The above linear increase, with the status quo winning, is the logical implication of Professor Wagner’s equations. Professor Wagner, however, does not calculate the growth in open information in this way. Rather, he drops Equation (2) entirely and uses a completely different formula, not presented in his paper, that results in the following table for the growth of open information over the first five iterative time periods:  

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>Period 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status Quo</td>
<td>1.5</td>
<td>3.0</td>
<td>6.0</td>
<td>12.0</td>
<td>24.0</td>
</tr>
<tr>
<td>(scenario 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less Control</td>
<td>2</td>
<td>3.3</td>
<td>5.6</td>
<td>9.3</td>
<td>15.4</td>
</tr>
<tr>
<td>(scenario 2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>More Control</td>
<td>1</td>
<td>2.3</td>
<td>5.4</td>
<td>12.7</td>
<td>29.6</td>
</tr>
<tr>
<td>(scenario 3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Professor Wagner’s results for the end of period 1, under the assumption that the amounts of the three different types of information are equal (to unity, for simplicity), follow directly from Equation (1) and the values of the $c_I$ and $c_{II}$ coefficients that have been assumed for each of the three scenarios. However, the remaining entries in the table are calculated from the following formula:

$$O_n = O_{n-1} + zO_{n-1} = O_{n-1}(1+z) \quad (3)$$

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15 Wagner, supra note 2, at 1021 tbl.5.

16 His numbers for the end of period 1 should be 1.5, 1.333, and 1.333, for the three scenarios, respectively. For some reason, Professor Wagner does not apply his incentives factor $z$, shown in Equation (2), to this first iteration period. See supra note 14.
where $O_n$ is the total amount of open information at the end of period $n$.\textsuperscript{17} Thus, instead of calculating the amount of open information produced during a given period and adding that to the total of the prior periods, he assumes that an amount equal the total amount existing at end of the prior period $O_{n-1}$ multiplied by the incentives factor $z$, is produced \textit{in the next single period}. This amount is then added to the total existing at the end of the prior period to get the new total. This is information begetting information with a vengeance—startling if correct. There may be some areas in which a particular work does spawn exponential growth, at least for a while, but Professor Wagner’s theory applies to all information, from your laundry list to the transistor.

\textsection{13} To test the plausibility of Professor Wagner’s claim, we may look at how Equation (3) relates the total amount of open information at the end of period $n$ to the amount existing at the end of the first period:

$$O_n = O_{n-1} + zO_{n-1} = O_{n-1}(1+z) = O_{n-2}(1+z)^2 = \ldots = O_1(1+z)^{n-1} \quad (4)$$

\textsection{14} Inserting the values of $z$ for Professor Wagner’s three scenarios, we see that

1. $O_n = 2^{n-1}O_1$ (scenario 1, status quo)
2. $O_n = (5/3)^{n-1}O_1$ (scenario 2, reduced control) \quad (5)
3. $O_n = (7/3)^{n-1}O_1$ (scenario 3, increased control)

\textsection{15} Thus, at the end of period 5, the amount of open information under scenario 1 is $2^4(1.5) = 24$; under scenario 2 is $(5/3)^4(2) = 7.72(2) = 15.44$; and under scenario 3 is $(7/3)^4(1) = 29.64$. These numbers exactly match those shown in Professor Wagner’s table.\textsuperscript{18} Although the value of $O_1$ differs for each scenario, that initial value becomes irrelevant with a sufficient number of time periods, because it is swamped by the exponential growth that this model assumes for information. As a matter of pure mathematics, there will invariably be a period $N$ beyond which scenario 3 will always outperform scenario 1, which in turn will always outperform scenario 2, regardless of their respective values of $O_1$,\textsuperscript{19} and the differences keep getting larger. Thus, Equation (4) necessarily implies that information begets information without the intervention of authors or inventors, except

\textsuperscript{17} In a private communication, Professor Wagner confirmed to me that this is the formula he used to create his table as well as the graphs shown in his article. E-mail from R. Polk Wagner, Professor of Law, University of Pennsylvania Law School, to Dennis S. Karjala, Jack E. Brown Professor of Law, Sandra Day O’Connor College of Law, Arizona State University (Aug. 13, 2005, 4:58 MST) (on file with author).

\textsuperscript{18} See Wagner, supra note 2, at 1021 tbl.5.

\textsuperscript{19} That is, there exists a number $N$ such that $(7/3)^{n-1}O_1 > 2^{n-1}O_1 > (5/3)^{n-1}O_1$ whenever $n > N$, regardless of any variance in $O_1$ from one scenario to another.
through the incentives factor \( z \), which having been increased or decreased once, should remain steady thereafter until a second change in control leads to another increase or decrease in incentives. It is, however, far from obvious how the amount of open public information can be said to be an exponentially growing factor multiplied by the amount of open information generated during the first period after a change in incentives.\(^{20}\)

\[ \text{Equation (4), which is the equation actually used by Professor Wagner to calculate the growth of open information, is precisely the formula for compound interest:} \]

\[ FV = PV(1+r)^n \]  

(6)

where \( FV \) is the future value, \( PV \) is the present value, \( r \) is the interest rate, and \( n \) is the number of compounding periods.\(^ {21}\) The future amount (value), \( O_n \), of open information at the end of period \( n \) is equal to the amount (value) present at the beginning \( O_1 \) multiplied by the \( n \)th power of one plus the growth rate.\(^ {22}\) Professor Wagner makes no effort to justify his use of the compound interest formula to make his calculations, nor the 100% growth rate he assumes for the status quo. (Recall that Equation (4) cannot be derived from Equation (2), which implies a linear rather than exponential growth for open information.) Mathematical modeling can be useful when the model approximates how the real world operates. Without justification that the compound interest model does actually approximate in some plausible way the operation of the real world of information growth, Professor Wagner has shown no more than that he knows how to calculate compound interest and that, given enough time, a higher interest rate always wins over a lower rate regardless of the amount of initial principal. All of the “mathematics” in his article is irrelevant to the crucial question of how much control intellectual property regimes should give to rightsholders and how that control relates to information growth.

II. Why “Open Information” Growth is Not Exponential

\( \text{¶17} \) The exponential increases assumed by Professor Wagner and shown in his table and the accompanying graphs do not fit any theory of incentives for the production of creative works that comports with the reality of intellectual property law. They simply cannot be correct. Effectively,

\(^{20}\) The factors 2, 5/3, and 7/3 for scenarios 1, 2, and 3, respectively, result from Professor Wagner’s assumption that open information grows by his incentive factor \( z \) multiplied by the total amount of open information existing at the end of the previous period. When the two are added to get a new total, the factor becomes \((z+1)\).


\(^{22}\) The exponent in Equation (4) is \( n-1 \) rather than Equation (6)’s \( n \) simply because we numbered the first iterative period “1” rather than “0.”
Professor Wagner has arranged his model so that the increased incentives operate on works rather than on creators, notwithstanding that his assumed changes in incentives come from giving creators more control over their intellectual property. He apparently assumes that increasing incentives in one period will not only increase the number of works created as a result of the new spur to authorship and invention but that the now increased supply of information will itself be the generator of yet new information in the succeeding period. This information self-generation is essentially independent of what authors and inventors are doing, because, as Equation (4) shows, the result after n periods depends solely on the initial amount of open information and the supposed incentive factor $z$.\(^{23}\)

\[\text{Equation (4) might bear some relationship to reality if the incentive scheme in question is iterative, in the sense that in every period we increase incentives by the incentive factor } z. \text{ Indeed, Equation (4) might then be true by definition if the incentive factor } z \text{ is itself defined as the increase in information that actually follows a change in incentives. See supra note 12. I, at least, have utterly no idea how we might do that, and nothing in Professor Wagner’s article suggests awareness that iterative increases in incentives, rather than a single increase at a given time, are necessary to make his theory even plausibly valid.}\]

\[\text{For simplicity, in referring to increases in the “number of authors” I mean to include both increases in the total number of people producing works of authorship and any increase in the amount of time spent on producing works of authorship by people who were already working as authors.}\]
Examples like this, however, do not show how much the total amount of open information has contributed to these pioneering advances or by how much these pioneering advances themselves actually increased the total beyond what would have occurred without them. After all, many of the people taking part in follow-on creations may well have been creating something else absent the pioneering advance on which they actually did base their work. Moreover, the pioneering advances, measured solely as a quantum of information, constitute a very small portion of the total. That a pioneering advance can lead to exponential growth in that field, at least for a time, in no way shows that all open information leads to such exponential growth. In fact, we would expect that most information will lead to little further growth at all. How many patents actually lead to commercially exploited products, for example, and how many of those that are exploited are pioneer in the sense of the transistor? How many films, novels, and paintings, not to mention speeches, lecture notes, and shampoo instructions, are seen, read, or admired to any significant extent at all, let alone serve as inspiration for exponential growth?

One can also posit examples in which the total amount of existing information has a negative effect on subsequent creator incentives, such as where a given field or genre becomes so crowded that later creators feel they have little to add and would not do well against so much competition. In short, while the amount of open information available to current creators on which to build new works probably does have some effect on creation incentives, we need a much more complete theory of the connection between open information and creator productivity. Professor Wagner, at any rate, has provided no basis for his implicit assumption that the total amount of open information grows exponentially by a power of one plus his incentive-change factor, in other words, like compound interest at a 100+% interest rate.

25 Wagner, supra note 2, at 1007.
26 For some extremes in the invention area, see generally TED VANCLEAVE, TOTALLY ABSURD INVENTIONS—AMERICA’S GOOFIEST PATENTS (2001); KENJI KAWAKAMI, 101 UNUSELESS JAPANESE INVENTIONS (1995) (describing inventions that are almost useless in a campy way that gives them an honored status).
27 Ray Kurzweil has developed a “law of accelerating returns,” which argues that both biological and technological processes involve positive feedback mechanisms that lead to exponential growth. Kurzweil, supra note 6. He concedes a law of diminishing returns for any given technology, but observes that, at least up to the present, a new technology has always come along to keep
We also know from nearly every sphere of activity that there is a law of diminishing returns. Professor Wagner’s theory starts where it should, by concentrating on creators. Giving creators more control, he posits, would likely spur them into more creative activity (and encourage others not currently working as authors or inventors to try their hands). That is a plausible assumption and one around which we can understand trying to build a general theory of information creation. Such a theory will depend on the relationship between creator control and incentives or increased output, and Professor Wagner understands this fully. The things growing. Thus, vacuum tubes give rise to a burst of technology that continues to grow at increasing rates until displaced by the transistor, followed by the integrated circuit, the computer, and finally computers designing computers. Isaac Asimov has also reviewed the growth of knowledge generally (not just technological knowledge). ISAAC ASIMOV, ASIMOV’S CHRONOLOGY OF SCIENCE AND DISCOVERY (1989). Asimov makes no explicit claim concerning the growth rate, but his account reveals that 2 million years passed between human bipedalism and the use of stone tools, another 1.5 million years to the harnessing of fire, 300,000 years to the advent of religion, and 180,000 years more to the development of human art, the bow and arrow, and oil lamps. Id. at 1–9. Then, starting about 8,000 B.C.E. things start to speed up, with new major developments every 1,000 years or so, then every few hundred, then roughly every decade until the Renaissance, then every few years, and so on until modern times when there are often ten to twenty major developments in a single year. See id. at 10–654. Obviously, Asimov sees scientific knowledge as having grown exponentially, essentially from the beginning. Neither Kurzweil nor Asimov, however, makes any attempt to relate the growth they observe to the incentives of intellectual property law. Moreover, both Kurzweil and Asimov see the exponential growth of major developments in human knowledge as having continued for millennia, during most of which society did not even have any intellectual property protection for creative individuals. For Kurzweil, it is always a key pioneering discovery that keeps the process going. While it is possible that these key discoveries would have come faster had intellectual property protections been stronger, we need some theoretical or empirical framework before we go about increasing rightsholder control under intellectual property law, because we can lose as well as gain when we strengthen such rights. Moreover, Kurzweil’s theory applies only to the fruits of technology. He supplies no basis for saying that copyright subject matter, such as literature, art, and music, grows in the same way. Even the most casual observation would suggest that it does not. For example, while the novel was once a new form of literature, and cubism a new form of art, and they both inspired numerous later creators, I have seen no evidence that growth inspired by the novel or any given painting school, is exponential.

\(^{28}\) Wagner, supra note 2, at 1023 (“The analysis suggests that the production of open information is determined by the details of the control-incentives relationship. . . . [T]he impact of intellectual property policy proposals are best evaluated according to their effects on incentives, rather than on their perceived effects on the public domain or open information”).
question is how giving more control to creators over their creations can continue to spur production at an exponentially increasing rate. Indeed, nearly all of human experience suggests precisely the opposite, namely, diminishing returns: Once the supply of open information gets very large, further increases can contribute very little to the production of new information. In particular, any theory of creativity that depends on the total volume of open information available to creators to build on must recognize that efficiency cannot exceed 100%.29 Once that level is reached, the existing group of authors and inventors simply cannot produce any more than they do already, regardless of how much more new information we give them to work with, so all growth would have to come from increasing the number of working authors and inventors. Yet, without additional rounds of increased incentives, what reason is there to expect that more people will try their hands at the creative arts?

¶23 Professor Wagner might argue that the increased supply of new information will induce others to join by making it easier for them to be authors and inventors. But this, too, must sooner or later reach some limit. Some people could not create economically useful information (the main type with which our intellectual property laws are concerned) regardless of the size of the existing information base society gives them. There also may be significant limits on the demand side, both from transaction costs and from there being only so much time readers can devote to books or drivers can devote to operating autos with lots of nifty new patented bells and whistles.

¶24 Therefore, even granting the possibility that a larger supply of open information might increase productivity, every valid theory must predict some diminishing returns effect. Moreover, there is utterly no reason to think that an increase in creator control would affect the efficiency with which creators use open information. How creative persons discover, extract, and use open information to produce new works is a fascinating question but one about which we know very little. Giving a creator more

29 Professor Chiappetta has pointed out to me in a private communication that even though individual efficiency cannot exceed 100%, changes in the tools that creators use to produce or users employ to consume information can still drive the absolute output higher, even though all producers are operating at their capacity. E-mail from Vincent F. Chiappetta, Professor of Law, Willamette University, to Dennis S. Karjala, Jack E. Brown Professor of Law, Sandra Day O’Connor College of Law, Arizona State University (July 30, 2006, 11:21 MST) (on file with author). That is correct, but we are now considering the effect of a change in rightsholder control, and to do that we should hold other forces as constant as possible. Nothing in Professor Wagner’s theory depends on or includes consideration of improvements in information production or consumption technologies.
control of her work product, by hypothesis, will induce her to work harder (or to switch into the creativity business), but how does that increase her natural talents for using the existing base of open information? Finally, even under Professor Wagner’s theory, correctly applied (that is to say, additively), the status quo results in the production of 1.5 units of open information in his first iteration period, while the increased control scenario gives only 1.33. Consequently, to the extent the amount of open information does make creators more efficient, we should still do better under the status quo than we do under Professor Wagner’s increased-control scenario, because they have more open information to work with after the first period and the lead continues to build linearly thereafter.

III. TOWARD A REAL-WORLD THEORY OF INCENTIVES

A reasonably accurate model of the growth of open information would be far more complex than Professor Wagner’s. Staying as close as possible to Professor Wagner’s notation, let us return to Equation (1):

\[ O = c_{i1} + c_{i2} + i_{iii} \]  (1)

We can now try applying Equation (1) to a few examples to see how we might adjust the model to bring it somewhat closer to reality.

A. Expiration of Rights

Equation (1) seems to miss an important contributor to the public domain (and therefore to open information)—namely, those works whose copyrights or patents expire in the given period. Because we are interested in the effect of changes in incentives for the creation of works, however, we can perhaps treat this factor as a constant for periods of equal length, because the number of works falling into the public domain, in the short-term at least, is determined by the number of works that were created at various times in the past, before the new incentives went into effect. Of course, if incentives do increase the number of works created and we wait long enough, those works will enter the public domain and the “constant” in the equation representing their contribution will begin to increase. In the short term, however, the differences in contributions to open information from the number of works entering the public domain due to copyright and patent expiration may not be significant from one period to the next.\(^{31}\)

\(^{30}\) In fact, Professor Wagner concludes that the more-control scenario results in only one unit in the first period. See supra note 14. That, too, we place by the way.

\(^{31}\) Patents, of course, expire much faster than copyrights and presumably should be accounted for somehow. Doing so, however, would require our specifying
B. Interdependent Variables

The bigger problem with the model underlying Professor Wagner’s Equation (1) is that all of the five variables in Equation (1) are themselves dependent on the incentives factor $z$, which in turn makes them dependent on one another. The result is very complex and nonlinear mathematics. Under Professor Wagner’s analysis, for example, increasing incentives by giving stronger controls to the copyright (or patent) owner reduces the constants $c_I$ and $c_{II}$ in Equation (1). However, by hypothesis, increased incentives will also increase the supply of Type I information $i_I$ and that, under Professor Wagner’s theory, automatically increases the indirect Type III information $i_{III}$ generated by the Type I information. If the decrease in $c_I$ is counterbalanced by an increase in $i_I$, that together with the increase in $i_{III}$ would imply an increase in the amount of open information. (In Professor Wagner’s perfect control scenario 3, $c_I$ is zero, so all of the increase has to come from $i_{III}$.) However, increased control should also reduce $c_{II}$, and that may not be offset by an increase in $i_{II}$. That is, increased control by the rightsholder leaves less of her work free for building new works by others, which in turn can reduce $i_{II}$, the new derivative works based directly on $i_I$. This reduction in $i_{II}$ can directly reduce $i_I$ as well, because, as Professor Wagner realizes, many new works that are derivative from Type I information will themselves be eligible for intellectual property protection.

The result under a correct application of Professor Wagner’s model, therefore, is a bit messy, to say the least. It is insufficient simply to multiply Equation (1) by a factor $z$ representing a change in incentives and start calculating, even if one calculates correctly (that is, additively) from then on. All five variables in Equation (1), namely, $i_I$, $i_{II}$, $i_{III}$, $c_I$, and $c_{II}$, are themselves functions of $z$ that do not move in the same direction when $z$ (i.e., incentives or control) is changed. This is a problem likely to bedevil any mathematical model of human behavior. If we knew even the general shape of these functional dependencies, we could perhaps draw some general conclusions, but we do not. One thing seems clear, however: Whatever the character of these variables, one cannot calculate growth or diminution of the amount of open information by simply iteratively multiplying a constant factor, however that factor is derived, over and over again.

the time period in question more precisely, so that we know how to bring the 20-year patent term into the model.

32 The model underlying Equations (1) and (2) is not the compound interest model Professor Wagner uses to do his calculations. See supra text accompanying note 15.

33 Id. at 1018 n.97.

34 See supra note 10 and accompanying text.
C. A Case Study: Term Extensions

A simple example might help bring the difficulties into the open. Consider the recent extension of the copyright term in the United States, from life plus fifty years to life plus seventy years, retroactively covering works published after 1922. While it is doubtful that this term extension had any effect on creation incentives in the real world, there is a theoretical possibility that incentives did increase slightly. Under Professor Wagner’s analysis, that is enough to make it a positive contributor, in the long run, to the amount of open information available to our society. But consider what the term extension actually brings about: By hypothesis we will assume that it results in some (small) increase in new works due to the now higher incentive to create new works. That should increase Type I information slightly, and under Professor Wagner’s theory it should also increase Type III because some Type III always develops from Type I. However, for twenty years (starting in 1998) there will be effectively no additions to the public domain due to expiration of copyright on previously protected works. That means that works that otherwise would have been available to current authors as a basis for creating new works

Sonny Bono Copyright Term Extension Act, P.L. 105-298 tit. I, 112 Stat. 2827 (1998) (codified at 17 U.S.C. §§ 108, 203, 301–304). For works published after 1922 and before 1964, the term was extended only if copyright in the work had been validly renewed in the twenty-eighth year after publication. 17 U.S.C. § 304(b). Works published after 1963 and before 1977 automatically have their initial 28-year terms renewed by 67 years, for a total of 95 years. Id. § 304(a)(2). Works published after 1977 have a term of either life plus seventy years (individual authors) or ninety-five years (entity authors), and renewal for them is not an issue. Id. § 302.

In his dissent in Eldred v. Ashcroft, Justice Breyer observed:

Regardless, even if this cited testimony were meant more specifically to tell Congress that somehow, somewhere, some potential author might be moved by the thought of great-grandchildren receiving copyright royalties a century hence, so might some potential author also be moved by the thought of royalties being paid for two centuries, five centuries, 1,000 years, “‘til the End of Time.” And from a rational economic perspective the time difference among these periods makes no real difference. The present extension will produce a copyright period of protection that, even under conservative assumptions, is worth more than 99.8% of protection in perpetuity (more than 99.99% for a songwriter like Irving Berlin and a song like Alexander’s Ragtime Band).


(more $i_i$) remain subject to authorial control, and the transaction costs associated with licensing those still-protected works will reduce the number of new works of this type that are created, in comparison to what would have been created without the term extension. This has to count as a reduction in $i_1$ (to the extent of new material) and $i_3$ (to the extent the information contained in the still-protected work fails to get further disseminated and discussed). When one thinks of the number of high schools or local drama groups that would like to perform *Showboat*, a single 1923 work that was “saved” from the public domain by the Sonny Bono Act, it is difficult to imagine that the purely theoretical increase in incentives from the Act can overcome new $i_1$ the $i_3$ losses that continued protection engenders.\textsuperscript{38}

§30 What about a purely prospective term extension—one that increases the copyright term only for works created after the effective date of the extension? Then we do not have the problem of immediate loss of new derivative works and lower cost performances that we suffer from the actual, retroactive term extension. If a prospective extension of the copyright term from life plus fifty years to life plus seventy years does increase creation incentives, even if only slightly,\textsuperscript{39} Professor Wagner’s theory predicts that, in the long run, it will lead to more open information. This case is much closer but still not airtight. If incentives for new works increase, we will get more Type I information and thereby more Type III information. We do not start losing anything under this prospective system for at least fifty years, when the new copyrights otherwise would have begun to expire. From then on, however, there will be a rolling period of twenty years in which fewer new derivative works will be created, because of the continuing control over the underlying works that, without the term

\textsuperscript{38} In a private communication, Professor Wagner agrees that I apply his theory correctly here, if the term extension does increase both control and incentives. He thinks that the term extension may increase control but most likely does not increase incentives. If the incentive effect is indeed very small, he believes, the retroactive term extension effected by the Sonny Bono Act was a major policy error, because it will take too long for the small incentive effect to overtake the huge immediate loss from the extended term for existing works. E-mail from R. Polk Wagner, Professor of Law, University of Pennsylvania Law School, to Dennis S. Karjala, Jack E. Brown Professor of Law, Sandra Day O’Connor College of Law, Arizona State University (Aug. 11, 2005, 12:30 MST) (on file with author). His mathematics purport to show, however, that eventually even this most disastrous piece of recent copyright legislation will work out for the best, if we just wait long enough (assuming the extension does increase creation incentives by some nonzero amount).

\textsuperscript{39} This incentive would not include, of course, the “incentive” discussed in the previous paragraph that comes from believing that future term extensions will also be retroactive.
extension, would now be in the public domain and freely available to new authors as building blocks. If the increase in incentives is very slight, we will probably not get enough new information in the first fifty years (and thereafter) to overcome the loss of new derivative works that begins in fifty years. In any event, it is clear that this loss of new derivative works must somehow get taken into consideration in the analysis.

D. The Public Domain as an End in Itself

¶31 This discussion helps focus on another significant problem with Professor Wagner’s model—the assumption that increasing the size of the public domain is an end in itself. Professor Wagner focuses on information not subject to a copyright owner’s control. He does this, admittedly, not because that is his goal but because that is the goal he attributes to public domain advocates. The overall social goal, however, is not simply to get works into the public domain for the sake of the public domain. The public domain has real economic value to the public in that works no longer subject to copyright are generally more broadly available and for a lower price. If this were all there were to it, “the larger the better” might be the watchword. But, as discussed above, the public domain also is the source of many new works—in copyright terms, derivative works—that themselves are protected by intellectual property law. The creation of these new works does not immediately increase the size of the public domain, but that does not mean they are without interest for those who seek to build and maintain a vibrant public domain. A film based on a public domain story, such as many films from the Disney company, will be copyright-protected, but no public domain advocate of whom I am aware questions the potential value of such works to the growth and development of our culture. The goal, then, is the creation of works desired by and available to society, not just the total amount of freely available information. The question is how to optimize the underlying factors that affect achievement of this goal, given that many of them pull in different directions.

40 Wagner, supra note 2, at 997 & n.8. He goes on to say that the best way to determine the impact of a proposed change in intellectual property rules is to consider the change’s effect on incentives. Id. at 1023. I do not disagree with any of this. The problem is not with his assumption of a positive relationship between control and incentives, which he agrees requires examination. Id. The problem is that even if you do get the incentives right for the production of $i_i$, his mathematical analysis does not throw any light onto the growth of the public domain or indeed even onto the growth of $i_o$, because it does not account for the loss of derivative works not created due to the increase in primary incentives for $i_i$. Indeed, $i_i$ and $i_o$ likely move in different directions when creation incentives are changed. See supra ¶27. Yet, in calculating the growth of open information, he multiplies both by the same factor $z$. 
CONCLUSION

¶32 If creation incentives were unnecessary—that is, if authors and inventors would do their work regardless of the exclusive rights of copyright and patent—we would be better off with no regime of intellectual property protection. But we know that, without some degree of protection, we will get fewer works, as authors and inventors turn to more (economically) rewarding tasks. Once some works have been made available, others will try to use and improve upon them. That activity, too, leads to new and desirable works. Too much protection increases creation incentives but not enough to counterbalance the loss of even newer works that, but for the increased protection, would have been based upon earlier works. Unfortunately, this longstanding, basic intellectual property problem is not easily or simply amenable to analysis with the precision of mathematics.