

PUBLIC PURPOSE FINANCE: MODEL APPENDIX

This *Model Appendix* provides a formal Model of the government’s choice of lending versus spending, which is discussed in the third scenario in Part IV of the Article.¹ This Model deepens our understanding of public purpose finance (PPF) by allowing a closer consideration of the parameters guiding government’s decision to lend rather than spend, under particular circumstances. The Model is formal, but it is also friendly: the abstract math is accompanied by a numerical example, and the math itself is restricted to linear algebra. As in Part IV,² the exposition progresses through various stages: first by having government try to fund the investment through ordinary spending, then by seeing the challenges of that strategy, and finally by seeing how lending helps overcome these challenges.

A caveat before we begin. As mentioned in Part I,³ the Model is preliminary, in ways that will require development and revision. Tax rates are flat, rather than progressive, and government borrowing and lending is interest free. A subtler omission in the Model concerns the macroeconomic assumptions. When discussing government borrowing, I do not make explicit whether investors purchase bonds with existing cash, or whether the purchase is funded through new money creation.⁴ In a sense, this distinction tracks important debates between a “loanable funds” view of government borrowing and an “endogenous money” view.⁵ Implicitly, this Model incorporates different aspects of both views in ways that may prove inconsistent.⁶

Model Environment and Overview

The environment for this Model is a society with two demographic groups: Group α and Group β . This Model has three time periods (t_0 through t_2).⁷ The baseline period (t_0) provides baseline levels of pre-tax income for the two groups (I_α and I_β). For example, imagine Group α earns \$90 whereas Group β earns \$10, such that national income ($I_\alpha + I_\beta$) is \$100. The Model does not specify the number of members for each group, so group per capita incomes (total income/number of group members) remain unspecified. For ease of exposition, assume the two groups are roughly equal in size, but Group α has demographic and political majority, so that it drives political decisions. In the back of our minds, we can also imagine Group α has higher per capita income than Group β . For example, where Group α and Group β have 60 and 40 members respectively, their per capita incomes are \$1.5 for Group α and only \$0.25 for Group β ($=\$90/60$ and $\$10/40$).

The baseline period also provides baseline levels of taxation. First, the legislator determines the amount of tax revenue that needs to be raised (TR_t), for example, \$20. Then, the

¹ See Nadav Orian Peer, *Public Purpose Finance: The Government’s Role as Lender*, 83 LAW & CONTEMP. PROBS., no. 1, 2020, at 115–18.

² See *id.*

³ See *id.* at 104.

⁴ This is generally the case when bonds are purchased by commercial banks and broker-dealers, and funded through new bank deposits or repurchase (repo) agreements.

⁵ See generally RANDALL WRAY, MODERN MONEY THEORY: A PRIMER ON MACROECONOMICS FOR SOVEREIGN MONETARY SYSTEMS (2012).

⁶ See discussion *infra* notes 8, 10.

⁷ Like many models of this sort, the number of calendar years in each period remains unspecified. As discussed below, the last period (t_2) in the Model can be understood as a longer period. Also, like many models of this kind, the Model assumes the last period marks the “end of time,” so no payoffs need to be considered beyond this point. This has the effect of leaving out of the analysis any longer-term benefits that would accrue from social investment.

legislator calculates the minimum level for an income tax rate (μ_t) that would raise this required revenue ($0 < \mu_t < 1$). Returning to our numerical example, with national income of \$100, and a tax revenue target of \$20, the tax rate will be 0.2 ($=\$20/\100). Group α 's after-tax income will be \$72 ($=\$90*(1-0.2)$) whereas Group β 's after-tax income will be \$8 ($=\$10*(1-0.2)$). To simplify, the income tax rate is flat (rather than progressive), but this aspect of the Model can be adjusted.

In the second period—the “decision period” (t_1)—our society is faced with an opportunity for social investment in Group β . This could be, for example, an opportunity to invest in housing with higher levels of economic opportunity. The investment project has a cost (C), for example, \$30, which government initially funds through the issuance of bonds.⁸ For simplicity, we will assume that the bonds are not interest-bearing. We initially assume that government spends on the investment outright, as opposed to lending bond proceeds to housing program beneficiaries. Throughout the second period, the two groups' incomes, required tax revenue, and tax liability all remain unchanged.

The final or “post-investment” period (t_2) is longer than the other periods, and is perhaps a couple of decades (though the number of years is left unspecified). In this period, we assume that the investment has indeed taken place at the end of the of the previous period and its benefits were fully realized. Thanks to the investment, the income level of Group β has now increased by a certain amount (D).⁹ We also assume that returns on the investment are greater than the initial cost ($C < D$), and likely by some considerable margin. For example, Group β 's baseline income of \$10 might have increased by \$60, to a total level of \$70, bringing national income to \$160.¹⁰ Another key feature of the post-investment period is that the bonds that government issued in the previous period mature, and need to be fully paid back. Because the government chose to spend on the project (as opposed to lending to Group β), funding for redeeming the bonds needs to be raised through a general increase in the income tax. To redeem \$30 in bonds (the cost of the project, C), the government's required tax revenue (TR_2) rises from the baseline level of \$20 to \$50. With a new national income of \$160, the income tax rate (μ_2) will have to rise to from 0.2 to 0.31 ($=\$50/\160). This means Group α 's after-tax income will drop from the initial \$72 to \$62 ($=\$90*(1-0.31)$) whereas Group β 's after-tax income will rise from the initial \$8 to \$48 ($=(\$10+\$60)*(1-0.31)$).

The key point to appreciate here is that Group α 's decision to support the investment project has led to a significant decrease in its after-tax income, about 14% ($=\$72-62)/\72). This loss makes it less likely that Group α would support the project in the first place: Group α would only support the investment project on the condition that its after-tax income is the same, or greater, than it was in the baseline period (the “Group α condition”). As it turns out, this condition is quite difficult to satisfy under the spending alternative, so despite the positive return of the project ($=\$60-\$30)/\$30$), the project would not go through.

The same Group α condition, however, will always be satisfied if the government chooses to lend to Group β in the second period instead of spending outright. In this case, Group β repays

⁸ If an endogenous money view of government borrowing were to be used in this Model, money creation through the act of government borrowing would create new flows of income in the economy. These flows of income—and their effect on income tax liability—would then need to be modeled more explicitly. *See supra* text accompanying note 5.

⁹ Where a loanable funds framework is used, investment in new government bonds would come at the expense of investment in other (private sector) projects. That opportunity cost would have to then be modeled explicitly (such that the net increase in national income would be smaller than D).

¹⁰ Note that the “D” term represents the gross return on the investment project, rather than net return (D-C) or the rate of return. To calculate the rate of return one needs to consider the number of years over which the net return is realized.

the loan in the third period, such that no tax increase is necessary. Given the increase in national income, the tax rate should actually drop, if spending remains at baseline level. For example, with national income of \$160, and spending of \$20, the tax rate in the post-investment period (μ_2) drops from 0.2 to 0.125, a 35% decrease. Group α 's after-tax income would rise from \$72 to \$79, a 9% increase. Assuming profit maximization, this increase in after-tax income provides Group α with an incentive to support social investment ex ante, as long as it is made through lending.

The shift from spending to lending requires, however, an additional step: verifying whether social investment is still beneficial to Group β . This question did not come up under the spending alternative discussed above, because the only cost borne by Group β came in the form of greater-tax liability in the third period. By definition, however, that increased tax liability was only a fraction of income gains to Group β . Thus, under our assumptions, social investment through spending is always beneficial to Group β . With the shift from spending to lending, this no longer holds. Group β not only increases its tax liability, but also has to repay the loan, and the calculation of its tax liability is based on its entire income (there is no deduction for payment of loan principal). Accordingly, the Group β condition for social investment is that the third period income—after tax and after loan repayment—would still be greater than baseline income (the “Group β condition”). This condition can be satisfied rather easily, except in cases where the net return on investment (D-C) is relatively low, and the tax rate is relatively high (this is explored more rigorously below). In our numerical example, we take \$70 in third-period income and subtract \$8.75 in tax liability ($=0.125*\70) and \$30 in loan repayment, arriving at final income of \$31.25. This still represents a substantial increase from baseline after-tax income (\$8), though admittedly, far below the increase in after-tax income that would have been available if the spending alternative had a political majority (\$48).¹¹

With this description of the Model environment, we now turn to Model equations.

Baseline Period (t_0)

Group α and Group β are each provided with baseline incomes of I_α and I_β respectively, where $I_\beta < I_\alpha$. The legislator requires an amount TR_0 in revenue, which it raises by a flat income tax at a rate of μ_0 , such that:

$$(1) TR_0 = (I_\alpha + I_\beta) * \mu_0$$

Decision Period (t_1)

The opportunity for social investment in Group β is presented, requiring an outlay of amount C ($0 < C$) in the current period. That investment would raise Group β 's income by a greater amount (D) in the post-investment period ($0 < C < D$). To fund social investment, the government borrows amount C, also to be paid back in the post-investment period. For simplicity, we assume borrowing is costless. Initially, we also assume that the government makes the investment through outright spending (rather than lending to Group β members).

Post-Investment Period (t_2)

In this period, the amount of revenue that the government needs to raise increases: the government maintains its spending from the previous period, and additionally raises taxes to repay its bonds. The total required tax revenue thus equals:

$$(2) TR_2 = TR_0 + C$$

¹¹ Note that the sums add up exactly (\$16.875) if it weren't to rounding (Group α increased tax liability under spending alternative = Group α decrease in tax liability under lending alternative + (Group β 's after tax income in the spending alternative – Group β 's after tax, after repayment income in the lending scenario).

Substituting TR_0 for (1) we obtain:

$$(2a) TR_2 = (I_\alpha + I_\beta) * \mu_0 + C$$

Thanks to the social investment, Group β 's income rises by amount D . Group α 's income remains unchanged. As a result:

$$(3) TR_2 = (I_\alpha + I_\beta + D) * \mu_2$$

Substituting (2a) for TR_2 :

$$(3a) (I_\alpha + I_\beta + D) * \mu_2 = (I_\alpha + I_\beta) * \mu_0 + C$$

Solving for μ_2 :

$$(3b) \mu_2 = \frac{(I_\alpha + I_\beta) * \mu_0 + C}{I_\alpha + I_\beta + D}$$

For social investment to be rational for Group α , its after-tax income in the post-investment period must be equal or greater than its after-tax income in the pre-investment period. The Group α condition is thus:

$$(4) I_\alpha(1 - \mu_2) \geq I_\alpha(1 - \mu_0)$$

Or simplified:

$$(4a) \mu_2 \leq \mu_0$$

Substituting μ_2 for (3b):

$$(4b) \frac{(I_\alpha + I_\beta) * \mu_0 + C}{I_\alpha + I_\beta + D} \leq \mu_0$$

After simplification:

$$(4c) D \geq C * \frac{1}{\mu_0}$$

This significance of the condition depicted in (4c) above is the following. For social investment to reduce the after-tax income of Group α (and thus help secure its support of investment ex ante), the return on investment (D) must be greater than or equal to the cost of investment multiplied by the inverse of the baseline tax rate. Note that the baseline tax rate would typically be a small fraction. In the United States, for example, the 2018 average effective federal tax rate was only 16% (0.16).¹² Thus, investment returns would need to be greater than cost by a factor of about 6.25 (=1/0.16). For example, returning to our initial numerical example, the project return would have to be \$187.5 (=6.25*\$30). Over a twenty-year period, this means a required rate of return (CAGR)

¹² 16% = \$3,329 billion in total revenue for 2018/\$20,500 in 2018 GDP. See Nadav Orian Peer, *Public Purpose Finance: Data Appendix*, 83 LAW & CONTEMP. PROBS., no. 1, 2020, at pt. III tbl.1 (providing the \$3,329 billion revenue figure) [hereinafter *Data Appendix*] (available at <https://scholarship.law.duke.edu/lcp/vol83/iss1/7>; follow "Data Appendix" link); *Gross Domestic Product*, FED. RESERVE BANK OF ST. LOUIS ECON. RESEARCH, <https://fred.stlouisfed.org/series/GDP#0> [<https://perma.cc/DWP6-3XAF>] (last updated Dec. 20, 2019) (providing the 2018 GDP figure). Parenthetically, it is an interesting question whether, for our purposes, the calculation of the average effective federal tax rate should include payroll taxes, which represent about a third of total revenue. See *Data Appendix*, at pt. III tbl.1. Payroll taxes are often accompanied by fresh liabilities for the government (for example, social security benefits), such that revenue is generally unavailable to cover investment costs. If payroll taxes are excluded from the calculation, the required rate of return for government to recoup social investment costs from future tax revenue jumps to nearly 12%. The calculation is the following. The average effective tax rate, without payroll taxes, is 10.5%=\$2,158 billion in revenue/\$20,500 billion in 2018 GDP. The required return on investment therefore equals \$285 (=1/0.105*\$30 project cost). Taking \$30 as present value, and \$285 as future value of a twenty-year period, the required rate of return (CAGR) is 11.91%.

of 9.6%,¹³ which is quite a high threshold. When the rate-of-return is positive, but does not meet this threshold, lending emerges as an important alternative to spending.

Lending Instead of Spending

Where government decides to lend instead of spend, the Group α condition is always met, as we now demonstrate. Because Group β repays for the loans, there is no need to raise any additional tax for bond repayment. Therefore:

$$(5) TR_2 = TR_0$$

Substituting TR_0 for (1):

$$(5a) TR_2 = (I_\alpha + I_\beta) * \mu_0$$

Given the increase in Group β 's income, its tax liability would rise, leading to a corresponding decrease in Group α 's tax liability. Substituting (3) for TR_2 :

$$(5b) (I_\alpha + I_\beta + D) * \mu_2 = (I_\alpha + I_\beta) * \mu_0$$

Solving for μ_2 :

$$(5c) \mu_2 = \frac{(I_\alpha + I_\beta) * \mu_0}{I_\alpha + I_\beta + D}$$

Now substituting (5c) for μ_2 in the in the Group α condition in (4a):

$$(6) \frac{(I_\alpha + I_\beta) * \mu_0}{I_\alpha + I_\beta + D} \leq \mu_0$$

Dividing by μ_0 ($\mu_0 > 0$):

$$(6a) \frac{I_\alpha + I_\beta}{I_\alpha + I_\beta + D} \leq 1$$

Under the lending alternative, the Group α condition is always met: D is by definition positive, so the denominator ($I_\alpha + I_\beta + D$) is greater than the numerator ($I_\alpha + I_\beta$).

Returns on social investment, however, must also be positive for Group β , meaning that its income ($I_\beta + D$) after loan repayment (C) and tax ($= \mu_2 * (I_\beta + D)$)¹⁴ in the post-investment period must be equal to or greater than its after-tax income in the pre-investment period ($= I_\beta(1 - \mu_0)$). The Group β condition for the lending alternative is thus:

$$(7) I_\beta + D - C - \mu_2 * (I_\beta + D) \geq I_\beta(1 - \mu_0)$$

After simplification:

$$(7a) I_\beta * \mu_0 - \mu_2 * (I_\beta + D) + (D - C) \geq 0$$

From here, the math becomes less elegant if we substitute μ_2 for (5c) to further simplify the Group β condition. There is, however, an easier way to appreciate the significance of this inequality. From (5c), we know that $\mu_2 < \mu_0$, because μ_0 is multiplied by a fraction that is less than 1 (by definition, $D > 0$). Furthermore, the expression in which μ_2 appears in (7a) is negative ($- \mu_2 * (I_\beta + D)$). For this reason, if we substitute μ_2 for μ_0 in (7a), the left hand-side of the equation is less likely to be greater than zero. Thus, the condition for Group β rewritten below (8), is relatively more constraining than the original.

$$(8) I_\beta * \mu_0 - \mu_0 * (I_\beta + D) + (D - C) \geq 0$$

¹³ Taking \$30 (C) at present value and \$187.5 as the required future value (D) that would allow government repayment of cost from taxes.

¹⁴ As discussed above, this takes into account the fact that Group β cannot deduct payments on loan principal (C) from its tax liability. Hence, tax liability is calculated as $\mu_0 * (I_\beta + D)$ rather than $\mu_0 * (I_\beta + D - C)$.

After simplification:

$$(8a) D \geq \frac{C}{1-\mu_0}$$

Note that this Group β condition under the lending alternative, looks somewhat similar to the Group α condition under the spending alternative in equation (4c). To reiterate:

$$(4c) D \geq C * \frac{1}{\mu_0}$$

The difference between the two conditions is that in the Group α spending condition (4c), cost is multiplied by the inverse of the tax rate, whereas in the Group β lending condition (8a), cost is multiplied by the inverse of one minus the tax rate. The key to appreciate here is that as long as μ_0 is less than 0.5, the Group β lending condition will be easier to satisfy than the Group α spending condition. That is the case because for every $\mu_0 < 0.5$, $(1-\mu_0) > \mu_0$. Note that μ_0 and $(1-\mu_0)$ in the denominators of the two conditions are both fractions. The greater the fraction in the denominators, the smaller the righthand side of both equations becomes when the C term is divided by that fraction. In turn, the smaller the righthand side, the more likely it is for D to satisfy the condition. Where $\mu_0 < 0.5$, the denominator in the Group β lending condition will always be greater than the denominator in the Group α spending condition, making the former condition easier to satisfy for a given size of D in relation to C.

For example, with an effective average tax rate at 16%, the denominator will equal 0.84 (=100%-16%) and the Group β condition can be met with a D (return on investment) that is only 1.2 times (the inverse of 0.84) greater than C. This threshold is very low compared to a factor of 6.25 (the inverse of 16%). Over a twenty-year period, a factor of 6.25 corresponds to a required rate of return (CAGR) of 9.6% to satisfy the Group α spending condition, whereas a factor of 1.2 requires a rate of return of only 0.92% to satisfy the Group β lending condition.¹⁵ Finally, recall that this the relatively more constraining version of the Group β lending condition than the original one written in (7a). Our discussion here applies *a fortiori* to the original version of the condition in (7a), such that that condition will be even easier to satisfy, as compared to the Group α spending condition.

¹⁵ With an effective average tax rate of 9%, the condition will be satisfied with a rate of return of only 0.5%. See *supra* note 10.