

Income-Distribution Dynamics with Endogenous Fertility

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In developing countries, fertility typically falls with education. For example, in Brazil, women with no education have three times as many children as women with ten or more years of education. Since children of the uneducated are less likely to become educated themselves, this threefold difference in fertility creates a major demographic force increasing the proportion of unskilled workers.

There is some evidence that the fertility differential between educated and uneducated women is greater in countries with more income inequality. Using data from 62 countries (88 country-years) on total fertility rates by women's educational attainment, we calculated fertility differentials in each country-year as the ordinary least squares (OLS) coefficient from regressing fertility on years of education. Fertility differentials between educated and uneducated women are typically greater in countries with high Gini coefficients of inequality (see Table 1).

One plausible hypothesis for why fertility declines with education is that educated women face higher opportunity costs of time spent rearing children because educated women command higher market wages. This hypothesis is consistent with greater fertility differentials in countries with more inequality and with the particular importance of women's education in determining fertility.

This paper examines the implications of combining the following three assumptions: (i) higher wages reduce fertility; (ii) children

of the unskilled are more likely to be unskilled; and (iii) skilled and unskilled workers are complements in production. A model incorporating these features implies that an initial increase in the proportion of unskilled workers will reduce wages of unskilled workers and, since this lowers their opportunity cost of raising children, will increase their fertility. Under the assumption that children of unskilled workers are more likely to be unskilled themselves, this will tend to increase the proportion of unskilled workers in the next generation. Therefore an initial increase in the fraction of unskilled workers produces a multiplier effect in subsequent generations, suggesting that improving educational opportunities for even small numbers of children of unskilled workers could lead to large changes in the steady-state distribution of skill.

A number of writers have previously explored the impact of differential fertility on the long-run population distribution (David Lam, 1986; C. Y. Cyrus Chu and Hui-Wen Koo, 1990; Samuel Preston and Cameron Campbell, 1993; Robert D. Mare, 1997). These papers use a Markovian framework in which fertility in each group and the probability that a child born to parents in one group will transit to another group are both independent of the distribution of the population across groups. In this Markovian setup, there is a unique steady state. However, as informally discussed by Robert Repetto (1978) and Nancy Birdsall (1988), fertility is likely to depend on wages, and thus on the distribution of population across groups, potentially creating positive feedback between inequality and fertility. We follow Momi Dahan and Daniel Tsiddon (1998) in formally examining a non-Markovian context in which fertility and incentives for education depend on the wage structure, and thus on the fraction of skilled workers in the population. However, Dahan and Tsiddon (1998) generate a demographic

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TABLE 1—FERTILITY DIFFERENTIALS REGRESSED
ON GINI COEFFICIENT, 1974–1995

| Independent variable | Dependent variable: OLS coefficient of total fertility rate on years of education | | | |
|----------------------|---|---------------------|--------------------|---------------------|
| | (i) | (ii) | (iii) | (iv) |
| Gini coefficient | 0.692** (0.179) | 0.640** (0.180) | 0.410* (0.164) | 0.254 (0.159) |
| Year | -0.006** (0.002) | -0.007** (0.002) | -0.004* (0.002) | -0.006** (0.002) |
| ln(GDP) | | -0.033 (0.022) | | -0.065** (0.019) |
| Latin America dummy | | | 0.169** (0.034) | 0.208** (0.034) |
| Constant | 12.592** (3.872) | 14.868** (4.145) | 7.957* (3.774) | 12.805** (3.929) |
| N: | 88 | 88 | 88 | 88 |

Notes: All regressions have country random effects. Standard errors are in parentheses.

Sources: Data on fertility are from E. Jones (1982), United Nations (1987, 1995), and G. Mboup and T. Saha (1998). Gini coefficients are from Klaus Deininger and Lyn Squire (1996). GDP is real GDP per capita in constant dollars (chain index) expressed in international prices, base 1985.

* Statistically significant at the 5-percent level.

** Statistically significant at the 1-percent level.

transition and Kuznets curve through transition dynamics in a model with a single steady state, whereas we explore the determinants of long-run inequality and show that the positive feedback between fertility differentials and income inequality may lead to multiple steady states. In particular, we show that, if the initial proportion of skilled workers is great enough, wage and fertility differentials between skilled and unskilled workers will be small, allowing the economy to converge to a steady state with low inequality. However, if the initial proportion of skilled workers is too low, inequality will be self-reinforcing, and the economy may approach a steady state with a low proportion of skilled workers and great inequality between the skilled and unskilled. Increasing the probability that children of unskilled parents will become skilled, for example, by expanding access to educational opportunities, will expand the basin of attraction for the low-inequality steady state and may eliminate the high-inequality steady state altogether.

I. The Model

Suppose the production technology is

$$(1) \quad Y = L_S^\alpha L_U^{1-\alpha}$$

where L_S and L_U are the number of skilled and unskilled workers, respectively. Assuming competitive factor markets, the wages of skilled and unskilled workers will be

$$(2) \quad w_S = \alpha \left(\frac{L_U}{L_S} \right)^{1-\alpha}$$

$$w_U = (1 - \alpha) \left(\frac{L_S}{L_U} \right)^\alpha.$$

Utility is given by $V = \ln(n) + X$, where n is the number of children and X is consumption. Raising each child requires a time commitment of φ , and the total time endowment of an individual is 1. Thus the budget constraint is $X = w(1 - n\varphi)$. This implies that the utility function can be rewritten as $V = \ln(n) + w(1 - n\varphi)$. The first-order condition for optimal fertility implies $n = 1/(w\varphi)$. Under the assumed quasi-linear utility function, higher wages lead people to have fewer children.

The number of children of unskilled and skilled workers is given by the number of adults in each group multiplied by the number of children per adult:

$$(3) \quad C_U = \frac{L_U}{\varphi(1 - \alpha) \left(\frac{L_S}{L_U} \right)^\alpha}$$

$$C_S = \frac{L_S}{\varphi\alpha \left(\frac{L_U}{L_S} \right)^{1-\alpha}}.$$

To complete the model, it is necessary to specify the process governing each individual's education decision. We seek a mechanism in which (i) educational decisions are responsive to the incentives provided by wage premia, and (ii) children of unskilled parents face higher costs of education than children of skilled parents, due to either differences in

home environments or capital-market imperfections. To capture these features, we assume that all children of skilled parents, along with a proportion θ of children of unskilled parents, need \tilde{L} units of time to become skilled, but that a proportion $1 - \theta$ of children of unskilled parents need \tilde{H} units of time to become skilled.¹ Children with a low time cost \tilde{L} of obtaining education do so when $\tilde{L}w_U < (w_S - w_U)(1 - \tilde{L})$ or, equivalently, when $1/(1 - \tilde{L}) < D$, where D , defined as w_S/w_U , is the anticipated wage differential when they become adults. Analogously, those with a time cost \tilde{H} obtain education if $1/(1 - \tilde{H}) < D$. To simplify the algebra below, we will define $L = 1/(1 - \tilde{L})$ and $H = 1/(1 - \tilde{H})$.

II. Income Distribution Dynamics

Define a steady state as a triplet (R^*, D^*, γ^*) , such that if the ratio of skilled to unskilled workers at time t is R^* , the wage differential would then be D^* , and if fertility and education decisions are taken optimally, the proportion of children of unskilled workers who become skilled will be γ^* , and the ratio of skilled to unskilled workers in the next generation will remain at R^* .

To solve for steady states, we will look for fixed points of $R_{t+1}(R_t)$. Note that if all children of skilled workers and a fraction γ of children of unskilled workers become skilled workers,

$$(4) \quad R_{t+1} = \frac{C_{S,t} + \gamma_t C_{U,t}}{(1 - \gamma_t)C_{U,t}} \\ = \frac{1}{1 - \gamma_t} \left(\frac{1 - \alpha}{\alpha} \right) R_t^2 + \frac{\gamma_t}{1 - \gamma_t}.$$

¹ If children of skilled parents require $\varepsilon < \tilde{L}$ units of time to obtain education, the steady states are identical. If a proportion of children of skilled parents also require \tilde{H} units of time to become skilled, but the proportion is less than $1 - \theta$, the proportion of children of unskilled who require \tilde{H} units of time, the basic story is similar but equation (5) becomes a cubic equation instead of a quadratic equation, and we no longer get closed-form solutions such as those of Proposition 1.

Setting $R_{t+1} = R_t$ in (4) implies that any steady state must satisfy the following quadratic equation:

$$(5) \quad \frac{1}{1 - \gamma^*} \left(\frac{1 - \alpha}{\alpha} \right) (R^*)^2 \\ - R^* + \frac{\gamma^*}{1 - \gamma^*} = 0.$$

There are three possible types of solutions to this equation to examine. (i) If the steady-state ratio of skilled to unskilled workers, R^* , induces a wage differential of exactly L , then γ , the proportion of children of unskilled workers who become skilled, will be less than or equal to θ , the proportion of these children with a low cost of education. (ii) Similarly, if R^* induces a wage differential of H , then $\gamma \geq \theta$. (iii) Finally if R^* induces a wage differential between L and H , $\gamma = \theta$. (The wage differential can never be expected to be above H or less than L , or else everyone or no one would become skilled, which would not be consistent with rational expectations.) We consider each of these three cases in turn.

PROPOSITION 1: *If and only if*

$$(6) \quad \theta \geq \frac{L\alpha - \alpha}{L^2 - \alpha L^2 + \alpha L} \equiv \theta_L$$

a steady state exists in which

$$(7) \quad [R_L^*, D_L^*, \gamma_L^*] \\ = \left[\frac{\alpha}{(1 - \alpha)L}, L, \frac{L\alpha - \alpha}{L^2 - \alpha L^2 + \alpha L} \right].$$

PROOF (Sketch):

From (2) it is possible to show that $R^* = \alpha/[(1 - \alpha)L]$ if and only if $D^* = L$. If $D^* = L$, the fraction θ of children of unskilled workers with a low cost of education will be indifferent between obtaining education or not, and hence $\gamma_L^* \leq \theta$. Substituting R_L^* into (5) yields the expression for γ_L^* in (7). Imposing $\gamma_L^* \leq \theta$ results in (6).

In this steady state, called the low-inequality steady state, the unskilled have more children than the skilled, but enough children of the unskilled become skilled in each period to maintain the skilled–unskilled ratio at R_L^* and thus the wage differential at L . An analogous argument can be used to derive the high-inequality steady state.

PROPOSITION 2: *If and only if*

$$(8) \quad \theta \leq \frac{H\alpha - \alpha}{H^2 - \alpha H^2 + \alpha H} \equiv \theta_H$$

a steady state exists in which

$$(9) \quad [R_H^*, D_H^*, \gamma_H^*] \\ = \left[\frac{\alpha}{(1 - \alpha)H}, H, \frac{H\alpha - \alpha}{H^2 - \alpha H^2 + \alpha H} \right].$$

In this high-inequality steady state, the wage differential is H , and hence all children who have a low cost of education become skilled, and (generically) some children with a high cost of education do so as well. The high-inequality steady state exists if a wage differential of H induces a sufficiently great fertility differential that, even if all children of unskilled workers with a low cost of education become skilled, the ratio of skilled to unskilled workers in the next period would still not be high enough to reduce the wage differential below H .

PROPOSITION 3: *The only other potential steady states have*

$$(10) \quad R_{\pm}^* = \frac{1 \pm \sqrt{1 - \frac{4(1 - \alpha)\theta}{\alpha(1 - \theta)^2}}}{\frac{2(1 - \alpha)}{(1 - \theta)\alpha}}.$$

PROOF (Sketch):

When the wage differential is between the two cutoffs, $L < D^* < H$, people will obtain education if and only if they have a low cost of doing so, and hence $\gamma^* = \theta$. Equation (10)

then follows immediately from solving for the roots of (5) and substituting $\gamma^* = \theta$.

Depending on the value of θ , the potential steady states described in Proposition 3 may or may not be admissible. It can be shown that if $\theta < \theta_H$, then the negative root, denoted R_-^* , is inadmissible, because $R_-^* < R_H^*$, and hence $D_-^* > H$. (For notational convenience we will henceforth refer to steady states simply by their value of R^* .) Thus R_-^* and R_H^* can never be admissible at the same time. Whenever R_-^* is admissible, it is stable and replaces R_H^* as the highest-inequality steady state. Moreover, it can also be shown that if $\theta < \theta_L$, R_+^* is inadmissible, because $R_+^* > R_L^*$, and hence $D_+^* < L$. Whenever R_+^* is admissible, it is an intermediate unstable steady state between two stable steady states, one with high inequality and one with low inequality. For θ greater than some $\theta_{critical}$, the term under the square root in (10) is negative, and hence neither root is an admissible steady state. Finally, it can be shown that $\theta_H, \theta_L \leq \theta_{critical}$. Hence for $\theta > \theta_{critical}$, only the low-inequality steady state exists.

It is useful to summarize how the dynamics depend on θ , the proportion of children of unskilled parents with a low cost of education, since it seems plausible that improving the education system or subsidizing education could increase θ . If θ is less than both θ_H and θ_L , only the high-inequality steady state, R_H^* , will exist. For $\theta > \theta_{critical}$, only the low-inequality steady state, R_L^* , will exist. For intermediate values of θ , multiple steady states may exist. For example, it is possible to show that if $L < 1 + [1/(1 - \alpha)]^{0.5}$, and $\theta_L < \theta < \theta_H$, there will be stable low- and high-inequality steady states at R_L^* and R_H^* and an intermediate unstable steady state at R_+^* . $R_{t+1}(R_t)$ will take the form in Figure 1. The system will converge to the high-inequality steady state if R_0 , the initial ratio of skilled to unskilled workers, is less than R_+^* , and to the low-inequality steady state if $R_0 > R_+^*$. Increases in θ , the proportion of children of unskilled parents who have a low cost of education, reduce R_+^* , and thus expand the basin of attraction from which the system approaches the low-inequality steady state. The model thus suggests that countries with R_0 just under R_+^* may

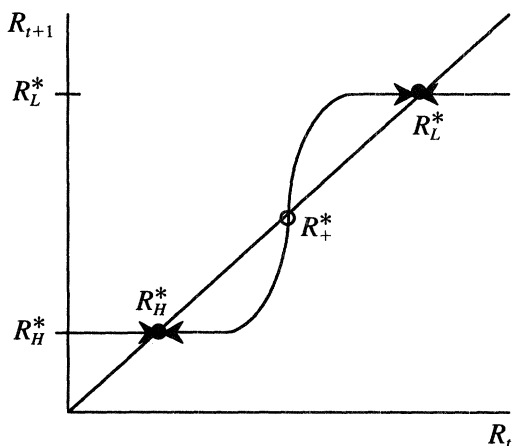


FIGURE 1. DYNAMICS WHEN $\theta_L < \theta < \theta_H$
AND $L < 1 + [1/(1 - \alpha)]^{0.5}$

face a brief window of opportunity in which small and temporary increases in θ can move them into the basin of attraction for the low-inequality steady state. As time passes, and R falls, larger or longer-lasting increases in θ would be necessary to move to the more equal steady state. The model also suggests that countries that reduce \check{L} or \check{H} , the cost of education for different segments of the population, can thereby reduce the corresponding steady-state level of inequality.

III. Limitations and Potential Extensions of the Analysis

The analysis can be generalized and extended in several ways. We have assumed a two-point form for the distribution of the cost of education. We conjecture that, with a more general distribution of costs of education, the $R_{t+1}(R_t)$ curve could cross the 45° line an arbitrary number of times, generating an arbitrary number of stable steady states. Generally, however, the response of fertility differentials to wages will make R_{t+1} increase more steeply in R_t than if fertility were exogenous. The steady-state distribution of skill will therefore be more sensitive to changes in parameters with endogenous fertility. If an additional 1,000,000 Brazilian children become educated, the unskilled wage rate will rise, raising

the opportunity cost of childbearing among the unskilled and reducing their fertility, which will further increase wages among the unskilled, creating a multiplier effect on inequality.

One limitation of the model is that the tractable quasi-linear utility function we use implies that fertility is inversely proportional to wages. This may be an acceptable approximation over the moderate wage levels characteristic of middle-income countries, but it is likely to fit less well at very low or high wages. At very low wages, wage increases may increase the number of surviving children, by reducing infant mortality and infertility due to disease and malnutrition. At high wages, further wage increases are likely to reduce fertility only modestly. Indeed, there is some evidence that fertility differentials by education are greatest among middle-income countries, and that the correlation between inequality and fertility differentials is primarily due to the middle-income countries.

Another limitation of the analysis is that we approximate relative wages as depending on the skilled-to-unskilled ratio of the population, rather than of market labor time, which also depends on the proportion of time each group spends in education and raising children. Dispensing with this approximation would make the problem more complicated: children's choices of whether to become educated would depend on their expectations of fertility of the skilled and unskilled when they become adults, since this would affect relative wages. Thus R_t would depend not only on R_{t-1} but also on R_{t+1} . We conjecture that most of the intuition would go through in such a model, but proof must await further research.

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