

## METAPHYSICS AND LAW

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*The dichotomy between questions of fact and questions of law serves as a starting point for the following discussion of the nature of legal reasoning. In the course of the dialogue the author notes similarities and dissimilarities between legal reasoning and philosophical and mathematical reasoning. In the end we are left with a clearer insight into the distinctive features of the adjudicative process.*

*Enter a Metaphysician, a Wrangler, a Physicist, and a Law Student.*

*Metaphysician:* I am concerned with the remarks of the Student of the common law over the question of axioms and questions of fact. I shall put it this way. (1) A man V says of a bird "It is a swan." The bird is black. (2) A man V says of a machine "It is a boat" or "This boat was exceeding the speed limit for boats." The machine does not touch water, it travels ten feet above it. (3) A man V says of another P "He intended to murder me." The idea has never entered P's head. Now there is a relation between the statement (A) "It is a swan" and the statement (B) "A swan need not be white" which is obvious. In the instance described (A) cannot be true unless (B) is true. This is less well expressed by saying that "(A) is true in the circumstances described implies that (B) is true." Now it must be remembered (A) is not an axiom but a statement of an actual state of affairs, for it is not a statement as to what constitutes a swan. Nor is it a statement of the sort "A thing of sort S1 could be of sort S2" or "A thing of sort S1 could not be sort S2." To call statements of the sort "This is a swan," "This is a boat," "This is an intention to murder," axioms suggests that these statements are statements which, if true, could not have been false. But they are not. To call them axioms obscures that even if they are in fact true, they might have been false. It obscures the fact that they are concerned with the actual . . . . To say the law is a calculus also obscures the fact. It obscures what I suggest is the central purpose of law. Lord Atkin's "You must take reasonable care etc. . . ." is indeed like an axiom. To point out the fact that in law undecided questions arise is no proof that it

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is not a calculus, no proof that it has a feature which is not present in a logical or mathematical calculus, is of the first importance. However, this should not be so presented as to obscure the fact that whether or not a question such as "Was there here negligence?" "Was there here manslaughter?" is a question of law, it is a question for the courts, and a question in which human beings, the courts, are seeking to gain a better knowledge of what actually happened. A procedure of the sort "Isn't this T1?" "Isn't this T2?" "Isn't this T4?" is appropriate to any statement of the sort "This too is of kind K" or "This is not of kind K," whether the statement is a priori or not. It is appropriate to "This is a cow," "This is a swan," "This is a sundial," as well as to "Two is a number." It is appropriate whether the statement is necessary or not. It is appropriate to "Is this a horse?" when "this" is a centaur, to "Is this a number?" when "this" is infinite. This procedure plays a part also for general statements "All K is K2," whether a priori or not. It plays a part because these statements depend upon statements of the sort "This is K," "This is K2." The procedure of parallel cases appropriate to "This is K1" should not be confused with the inductive procedure "This T1 is K1 and K2," "This T2 is K1 and K2," "This T3 is K1 and K2"; so "All K1 is K2," whether the connection between K1 and K2 is or is not a priori.

*Wrangler:* Certainly we must distinguish between questions of fact in the philosophic sense and the necessary truths of mathematics. We must distinguish between questions of law and questions of fact in the legal sense and questions of fact in the philosophic sense. It is crucial to comprehend what is contained in the legal question "Is this murder?" "Is this negligence?" In setting out the calculus-like nature of law I do not think that the Student has conflated these matters. It is true that it is difficult to find the axioms of a legal system. The self-evidence of certain legal propositions is no guarantee that they are axioms, nor is the generality of any legal statement. The Student did not suggest that the question "Is this murder?" or the statement "This is negligence" was an axiom. He said that the answer to these questions constituted theorems derived from axioms. This is quite explicit from the discussion of the limitation to "A and his heirs." Here are the axioms, "*Nullus heres viventis*" and "A man can

have an heir till the end of time." These are not statements of fact which could have been otherwise but logical possibilities. They are, in the contemplation of law, necessary truths with the characteristics and marks of Euclid's postulates and *modus ponens*. That these axioms are the product of human minds or the work of human beings or courts is no more an objection to their axiomatic nature than when we remember that much geometry was discovered when man wished to survey the lone and level sands or calculate the cubic capacity of the pyramids, or that Newton discovered the infinitesimal calculus to produce the laws of motion, or that the geometry of tensors and vectors is connected with Einstein's theory of relativity.

*Metaphysician:* Yes, this does raise some difficulty about the philosophic account of matters of fact and logical possibility and necessity.

*Wrangler:* Perhaps the habitual dichotomies necessary-contingent, a priori-empirical, analytic-synthetic, do not hold in law or indeed generally. I wish to suggest that the relationship between fact and law and law and fact is one of consonance. Lawyers say that such and such a concept is consonant with the facts, and such and such a case is consonant with some other case, where there is no equivalence between the two cases. The casual, not causal or logical, connection between the facts and the law may best be described by the term "consonant." Legal truths are thus consonant and may be compared with a metaphysician's talk of a reflective non-necessary truth. Shall we examine in greater detail this question of fact? I want to show that there are aleph facts and alpha facts at work in a legal system.

*Student:* Yes. We must distinguish between two senses of fact in the legal process. And then we must distinguish between questions of law, matters of law and questions of fact, matters of fact in the refined and distinct senses of alpha facts and aleph facts. To make this refined but not too recondite distinction we will first and foremost consider cases where there is a trial before both judge and jury. It is almost trite to start with the observation that the judge decides questions of law and the jury decides questions of fact.

*Wrangler:* Both aleph and alpha facts?

*Student:* Yes.

*Metaphysician:* But what is a question of law for the judge?

*Student:* Well, it is expressed often in this form: "Members of the jury you must take the law from me." It is a rule of law that "When the ancestor by any gift or conveyance takes an estate of freehold, and in the same gift or conveyance an estate is limited either mediately or immediately to his heirs in fee or in tail then always in such cases the heirs are words of limitation of the estate and not words of purchase." This is the famous Rule in Shelley's Case.<sup>1</sup> Again, "A charitable gift is for one of the following four purposes: the advancement of religion, the advancement of education, the relief of poverty and for other purposes beneficial to the community." Or, finally, "a descent cast tolls the entry into a right of action . . . ."

*Metaphysician:* I see, and what is an alpha fact?

*Student:* An alpha fact is one for the decision of the jury, who, having heard the evidence, decides a certain question such that the evidence alone is relevant to determine "Was there arsenic in the remains of the *corpus delicti*?" "Was the signature of the witness really the act and deed of the witness or was it a forgery?" "Did the horse with its rider jump the bales of straw?" "Was there a snail in the bottle or the decomposed remains thereof in the ginger beer when the plaintiff drank the same on such and such a day?"

*Wrangler:* You mean that when it is said that a case was decided upon the facts or the decision turned upon the facts, those facts are alpha facts?

*Student:* Yes. It is appropriate that you use the word decision turning upon facts and not judgment. These facts are essentially those facts to which it is appropriate for witnesses to answer, and which are within the province of induction.

*Physicist:* I can see that alpha facts are the sorts of facts with which a physicist is concerned. It is appropriate that even Popper<sup>2</sup> who denies that science is empirical and inductive, but asserts that it is hypothetical and deductive, nevertheless uses that part of the law which relates to jury trial as the analogue for scientific discovery. But what are aleph facts?

*Student:* Aleph facts are those matters which are left to the jury. They may be characterized in this sort of address: "Members of the jury, it is for you to decide whether on the evidence there

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<sup>1</sup> 76 Eng. Rep. 206 (K.B. 1581).

<sup>2</sup> K. POPPER, THE LOGIC OF SCIENTIFIC DISCOVERY (Eng. ed. 1959).

was a rider and a horse and whether the rider and the horse did as alleged jump the bales of straw, having found those alpha facts it is also for you to say whether this constituted negligence, or a breach of duty to take reasonable care. I have told you what constitutes negligence and you must decide whether this was negligence, whether the act complained of constituted negligence.”

*Wrangler:* The jury has to ascribe to the alpha facts a legal concept and having so ascribed to the alpha facts a legal quality this is the finding of an aleph fact?

*Student:* Yes. An example which makes the matter perfectly clear is contained in this passage from the judgment of Diplock L.J. in *Wooldridge v. Sumner*:<sup>3</sup>

To treat Lord Atkin's statement 'You must take reasonable care to avoid acts or omissions which you can reasonably foresee would be likely to injure your neighbour' as a complete exposition of the law of negligence is to mistake aphorism for exegesis. It does not purport to define reasonable care and was directed to identifying the person to whom the duty to take reasonable care is owed. What is reasonable care in a particular circumstance is a jury question and where, as in a case like this, there is no direct guidance or hindrance from authority it may be answered by enquiring whether the ordinary reasonable man would say that in all the circumstances the defendant's conduct was blameworthy.<sup>4</sup>

I think it is important to keep the distinction between alpha facts and aleph facts clear.

*Metaphysician:* Why? Is there here any distinction of philosophical importance?

*Student:* It draws a distinction between those cases where experiment, observation, looking, weighing, testing, measuring, subsequent observation and experiment establish facts empirically, and those facts which *reflection alone reveals*. The difference between "the Wasserman reaction was positive and so the patient has syphilis" on the one hand and "all the evidence is in, it has horns and cloven hooves etc. but is it a cow?" or the question "Is this a cow though it is a hermaphrodite?" "A horse though it is a centaur?" "A number although it is infinite?"

*Metaphysician:* Quite so. This is the distinction I have just made.

*Student:* What I am shewing is that the question of the type

<sup>3</sup> [1963] 2 Q.B. 43 (C.A.).

<sup>4</sup> *Id.* at 66-67.

which the Metaphysician has raised "Was there here negligence?" "Was there here manslaughter?" is a question by means of which human beings are seeking a better grasp of what happened. It is a jury question, and it is a question of aleph fact. By contrast, let me give an example of an alpha fact. Imagine that Fats Waller cross-examines Porphyria's lover in Browning's poem. The lover is charged with murder. He is asked "Where were you on the night of June the first?" He makes the following statement:

The rain set early in tonight, the sullen wind was soon awake, it tore the elm tops down for spite and did its worse to vex the lake. I listened with heart fit to break when glided in Porphyria. Straight she shut the cold out and the storm and kneeled and made the cheerless grate blaze up and all the cottage warm . . . and last she sat down by my side and called me. When no voice replied she put my arm about her waist and made her smooth white shoulder bare and all her yellow hair displaced, and stooping made my cheek lie there and spread o'er all her yellow hair, murmuring how she loved me—she too weak for all her heart's endeavour to set its struggling passion free from pride and vainer ties dissever . . . Surprise made my heart swell and still it grew while I debated what to do . . . I found a thing to do and all her hair in one long yellow string I wound three times her little throat around and strangled her.<sup>5</sup>

Stopping there we find a confession that he strangled her. The question for the moment is not that he murdered her which would be the answer to an aleph question but that he strangled her—an alpha question. Whether strangling constitutes murder in the circumstances is a jury question to be decided upon the law after the judge has directed the jury as to the nature of murder, say Coke's definition: "Where a person of sound memory and discretion—unlawfully killeth any reasonable creature in being—and under the King's peace—with malice aforethought, either express or implied."

I want to suggest that Popper's account of trial by jury does not fully identify two questions for the jury, though he does well to see that there is a difference between judgment by judges and verdicts by juries. However he conflates alpha and aleph facts and so does not draw the distinction between facts of which empirical evidence sufficed to shew that they happened, occurred or existed and facts where there is need for reflection, interpretation and a conceptual scheme in which they rest.

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<sup>5</sup> Browning, *Porphyria's Lover*, in *POETRY OF THE VICTORIAN PERIOD 177-78* (3d ed. J. Buckley & G. Woods 1965).

*Physicist:* Let us look at what Popper actually says:

The verdict of the jury (*vere dictum*—spoken truly), like that of the experimenter, is an answer to a question of fact (*quid facti?*) which must be put to the jury in the sharpest, the most definite form. But what question is asked, and how it is put, will depend very largely on the legal situation, i.e. on the prevailing system of criminal law (corresponding to a system of theories). By its decision, the jury accepts, by agreement, a statement about a factual occurrence—a basic statement, as it were. The significance of this decision lies in the fact that from it, together with the universal statements of the system (of criminal law) certain consequences can be deduced. In other words, the decision forms the basis for the *application* of the system; the verdict plays the part of a 'true statement of fact.' But it is clear that the statement need not be true merely because the jury has accepted it. This fact is acknowledged in the rule allowing a verdict to be quashed or revised.

The verdict is reached in accordance with a procedure which is governed by rules. These rules are based on certain fundamental principles which are chiefly, if not solely, designed to result in the discovery of objective truth. They sometimes leave room not only for subjective convictions but even for subjective bias. Yet even if we disregard these special aspects of the older procedure and imagine a procedure based solely on the aim of promoting the discovery of objective truth, it would still be the case that the verdict of the jury never justifies, or gives grounds for, the truth of what it asserts.

. . . .

In contrast to the verdict of the jury, the *judgment* of the judge is 'reason': it needs, and contains, a justification. The judge tries to justify it by, or deduce it logically from, other statements: the statements of the legal system, combined with the verdict that plays the role of initial conditions. This is why the judgment may be challenged on logical grounds. The Jury's decision, on the other hand, can only be challenged by questioning whether it has been reached in accordance with the accepted rules of procedure: i.e. formally, but not as to its content. (A justification of the content of a decision is significantly called a 'motivated report,' rather than a 'logically justified report.')

The analogy between this procedure and that by which we decide basic statements is clear. It throws light, for example, upon their relativity, and the way in which they depend upon questions raised by the theory. In the case of the trial by jury, it would be clearly impossible to *apply* the 'theory' unless there is first a verdict arrived at by decisions; yet the verdict has to be found in a procedure that conforms to, and thus applies, part of the general legal code. The case is analogous to that of basic statements. Their acceptance is part of the application of a theoretical system; and it is only this application which makes any further applications of the theoretical system possible.<sup>6</sup>

*Student:* Yes. Although Popper points out something illuminating here, he misses the nature of legal proceedings when

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<sup>6</sup> K. POPPER, *THE LOGIC OF SCIENTIFIC DISCOVERY* 109-11 (Eng. ed. 1959).

there is a question of fact to be decided. He ignores my trichotomy alpha facts, aleph facts and questions of law. A decision as to a matter of aleph facts is itself reasoned in just the same way as a judgment as to a matter of law is reasoned. When he says that judgments proceed logically he is saying something important. If he means that logic means only deductive logic, however, he is mistaken, for legal judgments proceed paraductively as well as deductively. And paradduction is also logical. His confusion is brought out when he says that the causal connections between the verdicts of juries or their decisions are psychological and not logical. When a jury has to decide an aleph fact that connection between decisions and fact is paradductive and is also logical. It is of course a matter for epistemologists and philosophers of science to say whether the connections between the existence of the external world, or the facts of the external world and our knowledge of them is a logical question.

*Metaphysician:* Quite so. The traditional moves made in philosophy bring out this feature. There is a tendency on the part of philosophers of science who support the hypothetico-deductive theory to consider that the theories or algorithms govern not merely the facts but also what are facts. It is quite clear that when a jury decides questions of alpha facts, it is presented with the question "What happened?" This is argued in accordance with our everyday conceptual scheme of those strange philosophical entities reality, mind, body, will, intention and so on. It is not confronted with the artificially limited interpretation put by the law which only comes into operation when the jury's decision concerns aleph facts. This does not mean to say that alpha facts are themselves free from philosophical, epistemological or metaphysical overtones.

*Student:* It is odd that even the hypothetico-deductive theorist should behave like the empirical and inductive theorist in accepting that facts are somehow brute, or can exist without cavil or speak louder than words.

*Metaphysician:* It is not so odd, for the difference between them really consists in this—that the hypothetico-deductive theorist seeks to establish that deduction alone guarantees our knowledge, while the empirical inductive theorist is concerned to claim that some principle of verification guarantees our knowledge. We have rid ourselves of the bondage of this confined view of logic by



acknowledging that paradiuction is also a logic. It is not a transcendental or intuitive argument, but, as the Student has pointed out, an explicitly rational procedure.

*Student:* I am also bringing out the point that a procedure of the sort "Isn't this T2?" "Isn't this T3?" "Isn't this T4?" is appropriate to any statement a priori or not, and is appropriate whether the statement is necessary or not.

*Metaphysician:* Yes, I see this. Between alpha facts and aleph facts there is a distinction which is reflected in my suggestion that the procedure is applicable to generalities such as "All K is K2" whether a priori or not. It plays a part, as I have said, because these statements depend upon statements of the sort "This is K1," "This is K2." However, the distinction I wish to bring out is this: that the procedure by parallel cases appropriate to "This is K1" should not be confused with the inductive procedure "This T1 is K1 and K2," "This T2 is K1 and K2," "This T3 is K1 and K2;" so "All K1 is K2" whether the connection between K1 and K2 is or is not a priori. This is what I wish to bring out in my distinction between alpha and aleph facts. There is a tendency to think that the paradiuctive argument or the case by case procedure is somehow inductive because it relates to facts and decisions, whereas it is concerned with concepts and with non-necessary truths, and is reflective. There is also a tendency to think that because it is concerned with concepts and because it is reflective it is appropriate only to that part of the judicial process or legal cases which is concerned with questions of fact, and having thus identified the appropriate procedure I characterize it by saying that juries have to decide aleph facts. I hope that this will at least make it clear to the lawyer and jurist how it is that Wisdom can say such a seemingly shocking thing—that the question "Is this negligence?" is a question of fact.

*Wrangler:* Having been impressed with what the Student has said about the relations between logic, mathematics and law, I would like to ask whether this diagnosis into matters of law, matters of aleph facts, and matters of alpha facts affects the Student's argument with the Doctor of the civil law.

*Student:* I do not think so for the following reasons. First of all, I have argued that the law and logic and mathematics rest ultimately upon paradiuction or the case by case procedure. With

this I trust the Metaphysician will agree. Then I suggested that the law is like logic and geometry in that certain given axioms could be propounded or postulated, and in accordance with certain moves, arguments and functions we could derive theorems from these axioms. An instance: the thirteenth century interpretation of the limitation to a donee, "A," being a life in being, "and his heirs." Another instance is the development by paradiuction between the years in the middle of the fourteenth century down to the late sixteenth of a series of cases which established the rule of law known as the Rule in Shelley's Case.<sup>7</sup> This seems to me to have something in common with the establishment of the rules of inference in logic and the commutative law in mathematics. Then I suggested that cases subsequent to *Shelley's Case* fall to be decided in several ways. Where the rule is applied, the courts use deductions from premises, that is the Rule in Shelley's Case. Where a case occurs the Rule may be interpreted, restricted or held not to apply. The most illuminating cases of this activity occur when we are dealing with rules of construction as in *Wild's Case*<sup>8</sup> or *Sibley v. Perry*,<sup>9</sup> rather than with rules of law as in *Shelley's Case* or *Purefoy v. Rogers*.<sup>10</sup> Insofar as there are areas in which there are axioms and rules, I suggest that a calculus may be operated by appropriate deductions. Here the analogy with mathematics and logic is strong, and may indeed be isomorphic. Where, however, it still remains appropriate, and this is in the majority of cases, that paradiuction is the proper procedure, nevertheless the calculus-like nature of the law remains. I also pointed out that in spite of undecidability problems, logic, mathematics and law all had consistency principles and all three could establish the consistency of the system by shewing how consistent a *part* of the system was. The law was shewn to be consistent, binding and logical, by pointing to clusters of cases which were binding, consistent and logical.

*Wrangler:* Is the inference to be drawn from this argument this alarming matter—as metaphysics has affinities and analogies with law, and as law has affinities and analogies with logic and mathematics, metaphysical questions like legal questions are like

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<sup>7</sup> 76 Eng. Rep. 206 (K.B. 1581).

<sup>8</sup> 77 Eng. Rep. 277 (K.B. 1599).

<sup>9</sup> 32 Eng. Rep. 211 (Ch. 1802).

<sup>10</sup> 85 Eng. Rep. 1181 (K.B. 1670).

logical questions? And metaphysical knowledge, logical knowledge and mathematical knowledge are the same?

*Student:* I would not say the same. They have family resemblances which intertwine, criss-cross and interrelate in such a way that those of us who say that mathematics and logic and law are knowledge and not nonsense, will say also of metaphysics that it is knowledge and not nonsense. We shall say that metaphysical questions are not verbal questions. We will understand that the problems which arise concerning the foundations of logic, mathematics and law resemble those foundational questions concerning metaphysics, epistemology and philosophy. We shall also be able to say that if mathematical arguments are rational and mathematical proofs conclusive, legal arguments are rational and legal proofs conclusive, and so on.

*Wrangler:* But how does your distinction between questions of law, and questions of aleph facts work? It seems to me that some remarks of Einstein are apposite here: "So far as mathematics do not relate to reality they are certain, and so far as they do relate to reality they are uncertain." Your question of law seems to me to be concerned with a purely conceptual matter, whereas your aleph facts, being a pastiche and melange of alpha facts and law, relate to reality.

*Metaphysician:* This worries me too a little. Consider this: Draw three triangles. This delta is an equal sided triangle and an equal angled triangle. This delta is an equal sided triangle and an equal angled triangle and this delta is an equal sided and an equal angled triangle, so all equal sided triangles are equal angled triangles. This is a process Johnson called intuitive induction. It is not argument by parallels. Scientific induction is concerned to establish a connection between two concepts which are two in the way cigarette smoking and cancer are two. Logical induction, mathematical induction and intuitive induction are concerned with a connection between two concepts which are two in the way equal sided triangle and equal angled triangle are two. Procedure by parallels involves only one concept. Does your distinction between law and aleph facts somehow distinguish between these senses of induction? I too am mystified for it seems that you are saying that questions of law, because they are calculus-like, are concerned with necessary truths in the sense in which it is often said that mathematical and logical concepts are necessary truths, whereas

the aleph facts seem more like the account I have given of those reflective, non-necessary questions of fact.

*Student:* I think I can answer both the Wrangler and the Metaphysician at once. My account of the law as a calculus involves certain artificialities which make it paradoxical to say that the law is heterodox and ultimate. But this is so. In the law the precedence of precedents, the hierarchy of authorities, the binding force of House of Lords decisions, the rules of pleading, the straight-jacketing features of legal language and definition make the ratiocination of the law curious and idiosyncratic. For these reasons law is like mathematics if the account Einstein gave of mathematics is correct. He said in the Princeton lectures that the mathematician was concerned to discover whether his theorems and so on were true insofar as they were derived from certain axioms and contained by certain arguments and functions; whereas a physicist was concerned to know whether the truths of his mathematical theorems corresponded with reality. Now a case is good law, a rule a good rule, if it is consistent with the rules of law or cases from which it is derived. Insofar as legal reasoning is not deductive and like the links of a chain, nor yet inductive and like the strength and length of a lever, but par deductive and like the legs of a chair which support a conclusion from certain arguments or ratiocinations, the truth of a rule or the soundness of a case is discovered with reference to its consistency within the body of other rules and cases. This is what Bacon meant when he said that the law was concerned with *de vero et de falso* and not with *de hono et de malo*, and this is what I argued *vis a vis* par aduction and the proof which supports a conclusion in my discussion with the Doctor of the civil law.

*Doctor:* Yes, I recall that Leibnitz too attributes something similar to the civil law in the Pandects.

*Wrangler:* Are you not saying that the values of a legal logic are the alethic modes and not deontic or epistemic?

*Student:* I am saying this, and more. I am suggesting that the correctness, rightness and truth of legal truths are tested by reference to the internal consistency of an informal system in the same way as the truths of mathematics and logic are tested with reference to the consistency of a formal system. I am also suggesting that a question of law is settled by operating a calculus. This operation consists in reflection upon the cases actual and

possible. It does not consist in the application of law to facts by the ascribing of certain legal concepts to facts as happens in the case of aleph factual questions.

*Metaphysician:* But is it not the case that aleph facts, decisions, are cases which form part of the calculus in a wider sense from the sources of law? The theorem, when applied to the alpha facts, constitutes the aleph facts, and are not these particulars of law, from which among others are derived the laws of laws, the *legum leges*? Does this not amount to the claim that the legal calculus is itself non-necessary and concerned with questions of fact which might have been otherwise?

*Student:* Yes, it is a theorem derived from axioms. There are possible cases also.

*Metaphysician:* Yes, but this removes law from the empirical inductive procedure of science. This does not place law in the a priori, necessary, deductive field of the mathematical and logical.

*Wrangler:* Analogy and induction have a place in mathematics and this does not take them out of the a priori necessary field.

*Physicist:* In science we use models and analogies, also hypotheses and deductions, as well as empiricism and induction.

*Student:* This brings us fairly and squarely into the sphere of foundational studies, and foundational studies are the concern of epistemology, and thus of philosophy.

*Wrangler:* Before we move into discussions of realism, intuitionism, finitism, conventionalism, etc., etc., scepticism, positivism and transcendentalism, we should consider in detail the use of cases, analogies, models and isomorphism as argument.

*Physicist:* The Metaphysician has suggested a distinction between the procedure by case, or paralogy, with other sorts of argument from particulars, parallels of cases, induction and what has been called intuitive induction and mathematical induction. In science we use analogies and models, pictures, theories and so on. The logic of scientific discovery and the nature of scientific explanation both use what philosophers of science call patterns of argument.

*Wrangler:* There are mathematicians who claim that in patterns of plausible reasoning in mathematics too we use analogies and induction, not mathematical induction either, but simple induction.<sup>11</sup> They argue that the sorts of arguments that jurymen and judges may use to come to different conclusions, which support

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<sup>11</sup> See G. POLYA, INDUCTION AND ANALOGY IN MATHEMATICS (1954).

but do not entail those conclusions, are analogous to the arguments used in probability.

*Physicist:* Yes, they distinguish between induction and plausible reasoning and we must distinguish between induction and deduction, physics and mathematics. The Student has brought out important features in the nature of legal argument by paradduction, but he has brought out with the distinction between law and fact, and in the refinement between law, alpha facts, and aleph facts, some features which are present I think in scientific matters: how, for example, facts are related to theories, and how theories are connected with calculi. "Busy old fool unruly sun," "All the rest the sun goes round," "The sun to me is dark." Are these facts brute facts, interpreted facts?

*Metaphysician:* Quite so. Alpha facts are somehow brute facts, uninterpreted facts: "He strangled her," "He forcibly seduced her," "The sun is like the bishop's bottom, large and round and hot." When a philosopher talks of facts and propositions, when he speaks of reality and the objects of the external world, he does not think that he is interpreting facts. He says of a question of fact that it is a question the answer to which if true might have been false and if false might have been true.

*Student:* When he says that "Is this negligence?" "Is this rape?" "Is this murder?" are questions of fact, does not the philosopher think that here there is interpretation or the setting up of facts in a conceptual scheme? Are these not set in a background of theory as Popper suggests decisions and verdicts of juries as to matters of facts are so set in the law? Do you not say that deciding whether this is negligence is the gain of a better grasp of what actually happened, the acquisition of a further understanding of the fact?

*Metaphysician:* Yes, by reflection and again reflection upon the cases actual and possible we may obtain a new grasp of what we did not see before, and this gain is a gain in our knowledge. It is not a psychological, but a logical activity.

*Physicist:* I am a little concerned about this. Quantum mechanics and the Copenhagen interpretation put me on my guard about the treatment of facts as brute, simple, "sensible to feeling as to sight." Do positrons exist? Is it a fact that positrons exist or is it proper only to talk of the concept of a positron? Yet I feel that

there is matter and a consistency in the nature of the world which the uncertainty principle does not disturb.

*Wrangler:* Are not the physicist's facts really mathematical equations, just algorithms? The physicist's theories merely correlate the facts with the calculi. Here I see an analogy with the Student's analysis of law. The objects seen by the Physicist with experiments are the alpha facts, the theories like that of relativity or classical mechanics are the aleph facts, and the calculi are the law.

*Physicist:* I think this is too facile, rather like Popper's analogy to facts and theories and the function of the jury I have described above.

*Student:* I think that we have underplayed the part of models and analogies in science. We have so far ignored the part played by analogies in formal systems. Did not Wang suggest that formalisation and isomorphism were but tight uses of the use of analogy and the comparisons which can be made with our intuitive ideas and messier concepts? The claim that we need formal identity to prove a case even in mathematics and logic is perhaps too strong. For a proof to be rigorous it may be the case that two calculi should be identical or be isomorphic. But cannot the standard of proof which we have to use to shew that imaginary numbers and infinite numbers are numbers suffice to prove the identity of calculi or that from two calculi certain results or conclusions may be obtained even if they are not isomorphic? So too with physical theories, cannot we compare models of them analogously and not isomorphically? This is what I thought some philosophers of science wished to shew. I said before that isomorphism is merely a formalised analogy. The benefits of formalisation may be two-fold. Formalisation may shew how rigid, clear, and easy are the points of correspondence between two systems. In a sufficiently rich system these identities become proofs, even rigorous proofs. But why should not the comparison of the informal which equally yields points of contact, comparison, and contrast, be considered proofs? They may not be formally rigorous, but nevertheless they may be proofs: not demonstrative reasoning, but plausible reasoning. After all, in courts of law we argue these matters every day.

*Metaphysician:* Certainly it is odd to talk of "proof," "establish," "shew," with reference only to induction and

deduction. A proof need not owe its strength to the links of a chain nor the length of a lever, it may be like the legs of a chair and support the conclusions. Conclusions may be derived from premises, they may be established by the cases or they may be supported by the premises. A rule we do well to remember is as strong as the cases which support it. We have noticed that peculiar questions cannot be proved by deduction nor by induction, yet it would be strange, even perverse, to say that we cannot prove them. "Is this a cow?" "Is this negligence?" "Is this a number?" "Is this a proof?"; these fall to be decided pareductively.

*Student:* A proof of evidence in a brief may relate to the establishment of alpha facts. A statement in a book on pleading or evidence, which tells us that in order to succeed in a case of trespass there must be proof that the plaintiff was in possession, concerns an establishment of aleph facts. I may have proved that the accused was not guilty (1) by shewing that on the night of June the first the accused was ten miles from the scene of the crime in Enfield committing adultery, when the accused was a monk; or (2) that killing is not murder in that the accused was insane; or (3) the accused was not guilty in that what was alleged in the indictment constituted no crime by shewing that adultery is no crime in English law though under the *lex Julia adulteriis* it was in Roman Law. These are different sorts of proofs.

*Metaphysician:* Quite so. There are other cases in which it is natural to say of someone that he proved something. Freud proved the existence of the Unconscious; Proust proved the existence of Love; Russell sought to shew that a mathematical proof must be perspicuous; Gauss proved that imaginary numbers were numbers, and so on.

*Student:* Again, in law we can establish a rule, prove a proposition, declare a rule of law. What we do is also said to follow sometimes inexorably, sometimes ineluctibly, sometimes of necessity. Are these euphemisms only psychological explanations of our procedures? Or is it the case that we use language naturally? Before a case is over we often feel that it was doubtful whether the accused was guilty, but now we are sure. We have gained a grasp of the facts and acquired a further understanding. I should also want to add we have acquired new knowledge. This is epistemological, not psychological merely.

*Metaphysician:* It is quite true that we must distinguish between epistemological questions and psychological questions. We must



distinguish between the questions how can we know and how do we know. Now what I want to ask here is the status of proof. Is it a can or do word? Why is a rigorous proof in any different position from an ordinary proof or an inductive proof? Is it in shewing us what we can know?

*Wrangler:* I do not know that it is not one way to know and so one way to prove that "An even number is always representable by the sum of two primes by running serially through the even numbers and decomposing each into primes." How did Aristotle find the syllogism? From arguments and patterns of plausible reasoning, then making rules about them. Of course there is a point that we cannot know intuitively all the prime numbers, or whether the number series is infinite. If the number series is infinite then there is no end to the number of even numbers and no end to the number of primes, so how can we know? It is a conceptual question we are considering. But will deduction answer our questions any better? As Wang again pointed out, there was a vain hope that when set theory was developed we would be able to answer foundational problems that arise in mathematics analytically. This turns out a vain belief. Foundational studies remain epistemological, and they are philosophical riddles not merely mathematical or logical problems.

*Metaphysician:* Quite so. This is what I have been trying to shew *vis a vis* all argument and knowledge.

*Student:* Yes. This is what I wish to shew as well. What I had in mind in shewing that so clearly a paraductive subject as the law was calculus-like, and so like mathematics and logic, was really a perverse way of shewing how mathematics and logic were themselves like the law, and how the fundamental and deep and foundational problems of both mathematics and logic resolved themselves into epistemological questions and were dissolved, if they were dissolved, by philosophical argument and analysis.

*Metaphysician:* Analysis is not a word I am too happy with. This smacks too much of the logico-deductive. The ultimate arguments in philosophy as in law, science, mathematics, logic, history, ethics and so on, are paraductive in your phrase, or case by case.

*Student:* Yes. I further attempted to shew that science was like law, history like law, the decisions of questions of fact like law,

that is aleph facts; and so there were resemblances between all these which shewed that all knowledge when studied in particularity, little by little, would yield the tale that all reasoning and all knowledge is derived from reasoning, and all knowledge is derived from reasoning which is paraductive, although there may be areas in which it is appropriate to talk of a priori, empirical, analytic, synthetic, necessary, contingent and so on. What is achieved by Polya is the doubting of the primacy of deduction in mathematics, what is shewn by Hesse is the doubting of the primacy of mathematical interpretation of physical data on the one hand and the primacy of the inductive claim that the facts must be the same before they can be thought of as particulars which yield general laws.

*Metaphysician:* Quite so. But neither sees in the use of cases the ultimacy of the ultimate argument; that ratiocination which lawyers have used as the supreme and final argument. It is the argument with which and by which counsel at the bar plead sometimes the general rule and its deduction from it when there is a point of law in issue, and at others how they refute inferences from the facts when an alpha fact is sought to be proved by evidence, whether it is the testimony of a witness or the proof of documents which are cases in which it would be appropriate to talk of induction. Finally it is used by judges when they sum up before, or direct a jury as to matters of aleph facts; that is, where it is usual to speak of fact and theory, reality and interpretation, but where a metaphysician like myself talks of matters of fact, question of fact. These foundational questions are in effect the raising of conceptual matters. How can we know an X? What is the nature of truth? What is the nature of proof?

*Wrangler:* Or what is the nature of number, infinity, mathematical identity and so on?

*Student:* Or what is the nature of law, what is the nature of justice right and wrong?

*Metaphysician:* Yes. These questions used to be talked of in the context of universals, or Plato's theory of forms, or Aristotle's essences. They revolve around the idea that we can give the sufficient and necessary conditions of an X or a cow or law.

*Student:* You would want to say that they are really the relations which are contained in Wittgenstein's notion of family resemblances, the interweaving, intertwining, criss-crossing,

overlapping of features that are common to some instances but not to others and certainly not to all with the features that others share with others but not with some and so on.

*Metaphysician:* Quite so. And these features are drawn out by paradduction by shewing cases where it would be appropriate to give such and such a description which corresponds to another description where the cases are analogous: "This is a swan;" "This is a number;" whether the number is rational, imaginary or infinite; "This is negligence;" "This is a positron or an atom or an element or a molecule or a particle;" or "This is real;" or "This is the unconscious" where we are talking of minds and so on.

*Physicist:* But I consider that the concept of a positron as the concept of quanta is a logical and not an empirical matter. The class of incontrovertibly true statements includes at least two kinds of different assertions: those such that their denials are self-contradictory, and those which are such that their denials, although consistent, describe nothing intelligible to think about. "All bachelors are male" is true in sense (1). "All bachelors weigh less than the sun" is true in sense (2). "No triangle is quadrilateral" is incontrovertibly true in sense (1). "No Beta particles or precisely determined energy can have precisely determinable co-ordinates" is true in sense (2). It may be that the suggested permanence of this last concept derives not from any positive conviction that there is a consistently intelligible alternative but from alternative views of demonstrable inconsistency. There is as yet no algebraically detailed, or an experimentally adaptable, answer to Heisenberg's challenge. Nothing in the past or present utterances of Born, Feyerabend and Vigier can claim here and now to be a working alternative. It is not enough just to note that since science evolved, this principle may change with it, for this is to note no more than physics is a body of empirical knowledge, and not a formally closed algorithm. No physical theory is incontrovertibly true in sense (1). Who would deny this? Not even Eddington at the zenith of his *a priorism*. Parts of our physical thinking however may now be incontrovertibly true in sense (2). Again, this is a conceptual formula, not a psychological one. That X is not thinkable is not another experimental datum. This affects everything that philosophers can say about X and its context. To say we do not know what a *perpetuum mobile* would be like is not like saying we do not know now what Venus's surface is like. Thus recognising

that a physical theory and its interpretation is never final *qua* logically closed is not the same as recognising, in addition, well developed alternatives of that theory, and of then asking which alternative is true.

*Wrangler:* This distinction between what is logically possible and what is pragmatically possible is important. I can give examples of abandoned proofs in mathematics; I can give examples of things which were said to be proofs, accepted as proofs, but which were not. They turned out to be false. I may have to dig them out from the limbo in which the obsolete and the false now rest. What are we to say of them?

*Student:* Yes, and what of cases in law that have been overruled, statutes repealed, judgments said to be false? When we think on these things and the connections with mathematics and logic which we have already discussed, the open-endedness of logic and mathematics, the effect of Godel's theorem, the liar paradox, the problems of foundational studies, the use of formalism and isomorphism and its relations with analogy, what is contained in following a rule, what is a rule, a proof, truth, and so on, we are left with little of so-called rigour in deductive logical systems. Little more that is than there is in induction and paraduction as arguments which prove.

*Wrangler:* Yet we do see differences.

*Student:* And likenesses.

*Metaphysician:* But what do these differences and likenesses shew? Do they prove anything? What have we gained after our discussion and how does that differ from what we knew before?

*Student:* We do not need to know everything to know something.

*Metaphysician:* *Tout comprendre c'est tout pardonner*, overstates the case.

*Student:* Yes, and in the law witnesses must tell the truth, the whole truth and nothing but the truth. To do this is no easy matter. It is a recondite, intricate, complicated process in which both counsel and judge have their separate roles, in which they ask questions, make inferences, relate facts one to another, discuss probabilities, argue the law, and so on. These difficulties are present whether the facts are microscopic or macroscopic. The physicist concerned with microscopic phenomena is inclined to consider that

his problems are conceptually and logically different from macroscopic phenomena. If this is so in physics it need not be so in finding alpha facts in law. How to relate the account of one observer with that of another may involve a theory of relativity, just as the physicist concerned with the problems of macroscopic and celestial motion needs an Einstein to resolve the metaphysical problems which arise in accounts of space and time. The observer from the moving train sees the falling apple differently from the observer standing on the railway line. To the former the fall describes a straight line, to the latter a parabola. To connect different accounts, to relate different lines of authority requires interpretation, comparison, and distinction. To see them and to shew them requires an understanding. Understanding requires argument. This is a facility as much concerned with following an argument as recognising a picture when it is a work of art, or the potentialities of a horse when it is to be handicapped. Such an ability and an activity employs in the manifold difficulties of a legal case the use of induction, deduction, paraduction. However, the processes of reasoning which might be used must not obscure the fact that one procedure alone ultimately supports other logics, that of deduction and induction. The place paraduction has in reasoning seems to me like the place Scheffer's stroke has in logic. The price we may pay for abandoning in practice deduction and induction may be too large to pay for metaphysical truth, but the law in its own way, and metaphysics in its ultimate way, must pay the price when we are asking about ultimate reasoning—an answer to the question "What is truth?" when it is the truth about reasons and knowledge.

*Wrangler:* I have heard much about paraduction, but what is it about this procedure, ratiocination, informal logic, ultimate argument, that is binding, forceful, compelling?

*Student:* Well, what is there compelling about following a rule in mathematics, being convinced by a proof and so on? These questions are related to foundational questions. They are questions about the fundamentals in disguise. We have shewn how the law and logic are related, interconnected, where they fit and square and where they are to be distinguished and so on. These seem to be fundamental to this question.

*Wrangler:* But do you think that the examination in detail, microscopically, of legal argument will reveal how the case by case

procedure is binding, how paraduction does convince? There has been much talk of the use of analogy, but what is the use of analogy?

*Student:* I have given examples of the use of paraduction and shewn how we can derive rules pareductively, how we can establish a part of our legal calculus pareductively, and I have argued that by proving part of our system to be consistent we can, as in mathematics and logic, shew how the whole is consistent. This is done in no small degree by the putting and countering of cases.

*Wrangler:* Yes, but what is the case?

*Metaphysician:* What can be usefully said about cases? How do they work in our language game? What role do they play in our calculus? The Wrangler's question must be answered as we would answer questions about time or number. Time may be blind continuum, a category, a coordinate, and so on; yet it may be that which is older than time, counted by chronometers and different from time counted by anxious, worried women lying awake; it may be older than generations. We have learned that it is not absolute. It is not a measuring rod counted by clocks or sundials or even dandelions; it may be relative to some observed phenomena simultaneously recorded by different observers. Now this informal account of time may also be used to shew that number is not the rigid mathematical entity of arithmetic only, but it may be related to music and poetry as in the way we talk of "tell me not in mournful numbers," etc. So too, cases must not be confined only to legal cases. We must be careful to distinguish between cases as used in induction, and in mathematical induction, and in deduction. The cases I have in mind are like legal cases and not like cases in induction or in mathematical induction. They are cases both actual and possible. They are cases deciding non-necessary questions of fact by reflection alone. They are distinguishable from induction in that subsequent testing, experimenting, looking, will not reveal any further facts or information, relations or constructs. It is as if all the evidence is in and yet there still remains a doubt.

*Physicist:* What do you hope to get out of the cases?

*Metaphysician:* I can tell you what I do not get out of the cases. I do not get out of the cases an inductive argument or a relationship which is causal. I do not get, this is a case of C1, this is a case of C2, this is a case of C3, . . . this is a case of Cn, so

this is a law: all C's are Gammas. I do not relate the argument causally in the way in which smoking and lung cancer are related—that is, relating two conceptions. I am concerned with deriving from the cases: “Is the case of C1 a case of C?” “Is this case of C2 a case of C?” when we are deciding something of the sort: “What is the nature of C?”

*Physicist:* Yes, this is indeed what is mystifying me. I understand that the law uses paradiuction to decide whether cases of nuisance and cases of negligence are cases of tort. I also understand that paradiuction is used to compare and contrast cases of contracts where there is no consideration with cases of equitable estoppel, thus shewing that contracts where there is no supporting consideration can be used as shields to an action, as defences.

*Student:* Yes, this is the case. There are others too. This is how Maitland describes the building of the doctrines of estates and can talk of the doctrine as establishing a calculus projected upon the plane of time.

*Physicist:* Quite so. The Student also suggests that paradiuction is analogous to the use of models and analogy in science. Now some accounts given by philosophers of science seem to support this view. Not all philosophers of science dismiss analogy as a mere use. What is unclear now is whether the metaphysician in drawing the distinction between cases and induction has diverged from his agreements with the Student about the role cases play in argument, the nature of cases and what it is we want out of them.

*Student:* I want the cases to present an analogy. I do not subscribe to the sophisticated view of some common law lawyers that all we can compare in a case are the *rationes decidendi*, and that the doctrine of precedents and the *stare decisis* rule apply only to the *rationes decidendi* of cases. Although in a way this formulation of the doctrine of precedents brings out a feature of how we use cases, we do not use the bare brute facts of one case and compare and contrast them to those bare brute facts of other cases. We do not use alpha facts of case 1 as points of contract, comparison, contrast, identity, equivalence, etc. with alpha facts of case 2. There is a well known way of deciding cases where the facts of the instant case are on “all fours” with the facts of a case which is a precedent. Where the instant case and the precedent cannot be distinguished, then the precedent, if it is an authority within the hierarchy, must be followed. The precedent is binding or persuasive.

We talk like this where the cases cannot be distinguished "on the facts" as the forensic expression goes. What is more often done, and what seems to me of interest to consider, is that part of the judicial process where aleph facts are used as the *point de rattachement*. Where the aleph facts contain the analogies, fact and interpretation constitute the analogy. This is not "loosely putting cases without laying any foundation of a ground or difference or reason" and "being put so scattered prove not" of which Bacon complains.

*Physicist:* Are you suggesting that Bacon's use of cases in legal writings and his new induction have a similarity?

*Student:* Yes, I think that this is clearly seen in Bacon's epistemology when we compare legal, philosophic, scientific, and historical writings. His insistence on particulars and finding from particulars, hidden axioms, such as the quiddities of heat, from which to induce higher axioms, is comparable with his desire to obtain the *leges legum* from the *placita legum*.

*Physicist:* This is interesting for what I want to shew is that the use of analogies and models in science have some analogy and point here. First, let me say that I was struck by the aleph fact and alpha fact dichotomy and I wonder whether the following procedure would not be too remote from what the law does in these matters. Consider the superficially observable similarities between waves of water, light and sound. In all three cases there are present motion, something transmitted indirectly from one place to another by hitting an obstacle, and a bending round obstacles. This suggests that the three processes are perhaps alike in more fundamental respects and in order to investigate this possibility we should look more closely at the one of the three about which we know the most, namely water waves. We postulate with Huygens that disturbance of one particle communicates with another particle in the same way a ripple causes disturbances in concentric circles, and by means of the elementary mathematics for harmonic motion we are able to represent the amplitude and frequency of the waves and to derive the laws of reflection and refraction. From this mathematical theory some rules of the process such as the quality of the angles of the incidence of refraction can be deduced. So far we have two sources of information to aid our constructions of theories for sound and for light—their observed properties and their observed analogies with water waves, and it is important to notice that both



of them appeal only to descriptions of "observable" events. We may define observation statements as those descriptive statements whose truth or falsity in the face of given empirical circumstances would be agreed upon by all users of English with or without scientific training. Let us also introduce the term "explicandum" for the set of observation statements connected with the phenomena we are attempting to explain by means of the theory, in this case the observed properties of sound or of light. All users of English might not of course notice the analogies between the three processes until they are pointed out, and up to this point they may have had no more significance than the fact that the fingers on the hand and the petals on a buttercup are similar in that they both have only five members. But where the analogies have been pointed out, no esoteric insight and no specifically scientific knowledge is required in order to recognise that they exist. It is not quite the same with the mathematical theory of water waves, for here some knowledge of trigonometry is required. But there is no difficulty in understanding the *terms* "height of water," "frequency of waves," etc., into which the mathematical symbols are interpreted. In this sense the mathematical system is "about [has its interpretation in terms of] observable events."<sup>12</sup> Even at this level of development of a theory of analogy and models, we find that the analogy consists in some sort of hidden features which physical objects possess and observation reveals. In Bacon's language, we find the hidden or latent qualities, not the simple forms. We shall see that when we construct analogies out of models we do so not to notice as the result of observation or experiment that particular states of affairs obtain, but that we may construct hypotheses out of the analogies. The analogy supports a theory rather than describes facts. In this view of science, the interest lies in the confirmation of hypotheses rather than in proving facts by induction. Of course the description of similarities and differences between two analogues is a fairly inaccurate, incomplete, and inconclusive procedure. Although we often feel some confidence in asserting the existence of a similarity and that some things are more similar to each other than to other things, we cannot usually locate discrete characteristics in one object which are positively and finally identifiable with, or differentiable from, those in another object. Still, the inconclusive

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<sup>12</sup> M. HESSE, *MODELS AND ANALOGIES IN SCIENCE* 149 (1963).

nature of the procedure is not fatal because we are not looking for incorrigible inductive methods, but only for methods of selecting hypotheses. Analogical argument is usually quite inconclusive, both for the usual inductive reasons and also because it may rest on incomplete implication or similarities. Nevertheless, as will be argued, it does present a method of hypothesis selection which is justified on at least some of the recognised criteria for such selection. We shall also see that even if it were possible actually to weigh the similarities between the explicandum and different models which are compared with it, this would not strengthen the analogical arguments according to any of its acceptable justifications, and so it is pointless even to seek to define the degree of similarity among the sets of characters. Polya tries to give a formalised account of patterns of plausible reasoning by using the logic of induction.<sup>13</sup>

*Metaphysician:* This brings out part of what I am suggesting. "How can we know?" is answered by the Physicist: "By the present use of analogies and models." Thus we can use analogies and models to discover hypotheses. In the similarities we see certain relations which hold. Here at least there is some likeness to my description of the case by case procedure. Where we differ is this: How does a scientist proceed when he has discovered a hypothesis? He wishes to confirm it or establish it. He then proceeds by experiment. He looks. A metaphysician cannot look except metaphorically. He can only think, reflect, cogitate, animadvert. He has no procedure open but to reflect upon what is already before him and is contained in the cases.

*Student:* And the lawyer too, the judge and the jury are in this position when they consider aleph facts or when the judge has to decide a question of law.

*Metaphysician:* It would never be right to say that a conceptual matter, a logical matter, was proved by subsequent experiment or observations. The facts can never prove a logical puzzle. Logic is independent of life, the facts, what is the case.

*Physicist:* Yet people say that Einstein's theory of relativity was proved by subsequent observation of certain astronomical phenomena.

*Metaphysician:* Only as a physical theory. Insofar as our concept of space and time as subsistent separate coordinates have been

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<sup>13</sup> G. POLYA, PATTERNS OF PLAUSIBLE INFERENCE (1954); see Hesse, *On Defining Analogy*, 60 PROCEEDINGS OF THE ARISTOTELIAN SOCIETY 79-100 (1959).

changed by Einstein, neither physics nor astronomy affect this metaphysical claim. Einstein was explicit about this in the Princeton lecture. Hence the aphorism already mentioned. "So far as mathematics relate to reality, they are uncertain. So far as they do not relate to reality, they are certain."

*Student:* And the law in this regard is like logic, mathematics, metaphysics. The facts and the events of the external world are merely the occasions for our reflections and concepts. Reality has only a casual connection with logic, neither causal nor logical. The claim is sometimes made that we cannot see these similarities and distinctions informally. However the history of English law shews that judges can and do reconcile and distinguish, without the aid of any formal system of logic or mathematics, or indeed any logic of induction. What is mystifying the claim that we cannot see, is the vagueness or ambiguity of the word "see," a word having overtones of observation, experiment, test, measure, and so on. In a sense it has empirical and inductive overtones. "Seeing," however, is used metaphorically and really imports the notions of classification and individuation, reconciling and distinguishing. These are not psychological but philosophical concepts. They are fundamental to the concept of mind, thought, and action. Some technique, some artificial process, some formal system, may help us to see more easily and quickly. It is not the case that without the formalisations that mathematicians and logicians talk of, we cannot see. I would rather say that without the formalisations it is not the case that we cannot individuate and classify or that we cannot distinguish and reconcile. The technique of *ratio decidendi* is a judicial way of giving us a technique to make us ready lawyers, precise lawyers. By *ratio decidendi* we have a facility, not a faculty, a capability not a capacity. The *rationes decidendi* of cases may represent the latent qualities in the simple forms or, in Bacon's language, the middle axioms derived from the first axioms. This is the element of hypothesis and the facts.

*Wrangler:* Theory is inseparable from science. Is the Student now conflating middle axioms, hypotheses in science, with aleph facts?

*Student:* No. Aleph facts are pastiches and melanges of law and fact which a jury decides. *Ratio decidendi* is something like the

statement of the judge in the *Case of Thorns*: "and the reason for all these cases is that he who is injured ought to be recompensed . . . ."<sup>14</sup>

*Physicist*: A philosopher of science would explain analogies insofar as they relate to induction, and analogies insofar as they relate to the mathematical theories with which a scientist is concerned, in these terms. The discussion following, which consists of quotation from Hesse, brings out the Metaphysician's distinction between analogies which illustrate a causal connection, and those where the argument from parallels is intuitive induction and mathematical induction.

Thirdly and finally, if we regard as valid the argument by analogy and models as essentially a transfer of causal relations between characters from one side of the analogy relation to the other, it follows that the interpretation of theoretical terms we have just given is *presupposed* in the argument, even if not explicitly referred to. For if there is a theory about the causal relations in model two, then the same theory holds for the relevant characters in model one, and hence for the explanatory theory being sought. To summarize the discussion so far; we have found some distinction between *formal* and *material* analogy to be crucial to the predictive use of scientific models, and in this chapter material analogy has been characterised by two types of dyadic relation, those of *similarity* and *causality*. Analogical arguments from formal models have been formulated in terms of *characters* which are independently observable but not also experimentally separable. . . . In connection with the justification of analogical arguments a distinction has been made between the *logical* problems of justifying inference from similarity and the *causal* problem of deciding whether the type of vertical relation implied in the analogy is acceptable as causal for either or both the analogues. Finally the meaning of theoretical terms has been elucidated in terms of the analogy relation defined.<sup>15</sup>

*Metaphysician*: When the Student talks of analogy and compares legal cases with Wang's suggestion that formalisation and isomorphism are the application of the method of analogy "used here," he is drawing to our attention that there is a logical relation between *similia* and *similibus*, between case and case. He is not using a causal connection, but a logical relation.

*Student*: Yes. This is why I suggest that the distinction between alpha facts and aleph facts is important. The doctrine of precedents is built upon the comparing and contrasting of aleph facts. Very rarely indeed are alpha facts used in paradiuction. These, I have said before and reiterate now, are cases which are said to be on all fours with another case. While it is true that some cases are

<sup>14</sup> 75 Eng. Rep. 1079 (K.B. 1543).

<sup>15</sup> M. HESSE, MODELS AND ANALOGIES IN SCIENCE 110-11 (1963).

decided like this, they are generally uninteresting. They are appropriately spoken of as cases turning upon the facts. Lawyers do not treat such cases as having a *ratio decidendi*. Even on the traditional analysis of *ratio decidendi* lawyers are concerned with alpha facts and rules of law. This again seems to me another argument which brings out the logical features of the law, and supports my calculus theory of law.

*Wrangler:* There are writers who say that every case has a *ratio decidendi*.

*Student:* Yes, but I think that this is mistaken. There are cases where the law is known but the alpha facts are in doubt. The judge, though more often the jury, decides what are the alpha facts, either on the balance of probabilities in a civil action or beyond all reasonable doubt in a criminal case. When these alpha facts are known, no difficulty arises in applying the law. Such cases frequently occur in the construction of holograph wills and cases in the criminal courts where the question is "Did Vacquier put the poison in the bottle of salts?" "Did the defendant publish the words complained of?" "Was the claimant really Tichbourne?" These are not questions of law. No citing of authors will help.

*Wrangler:* In the *Case of Thorns* which the Student cited, the judge cites the facts of cases, which are compared. These look like alpha facts; I lop fallows and stop up a stream and I unwittingly hit another with my staff, etc., etc. Here the judge says: "And the reason for all these cases is that he who is damaged ought to be recompensed." Surely this is declaring a rule of law directly from alpha facts. We are deriving a general rule from Bacon's simple forms, or his first axioms. It seems to me that this is a case of induction by enumeration only. There are jurists, among them American Realists and Scandinavian Realists, who talk of law as facts and consider that the law is inductively established by observation of the facts. For example, Cook says:

In the conclusion that a court never enforces foreign rights, but only rights created by its own law, I see nothing extraordinary. Indeed, if we examine into the meaning of the terms 'law' and 'right' as they are commonly used by judges and lawyers, I think we shall conclude that this way of stating the matter is the only satisfactory way. For we as lawyers, like the physical scientists, are engaged in the study of objective physical phenomena. Instead of the behaviour of electrons, atoms, or planets, however, we are dealing with the behaviour of human beings. As practicing lawyers we are interested in knowing how certain officials of society—judges, legislators,

and others—have behaved in the past, in order that we may make a prediction of their probable behaviour in the future. Our statements of the 'law' are therefore 'true' if they accurately and as simply as possible describe the past behaviour and predict the future behaviour of these societal agents. A statement, for example, that a certain 'rule of law' is the 'law of England' is therefore merely a more or less convenient shorthand way of saying that, on the basis of certain observations of past phenomena, we predict certain future behaviour of the appropriate English officials.<sup>16</sup>

How does the Student maintain the value of his distinction and the importance of the distinction between alpha and aleph facts in view of this approach?

*Physicist:* Indeed, when the Student suggests that there is no difficulty in seeing the similarities and differences between analogies or cases, he can hardly deny the validity of the argument from simple enumeration only. Even Bacon's argument arising from the counter-instance is not available, for there are no counter-examples or instances which are fatal to Aristotle's argument, for he was dealing with the repetition of the same cases. In the case of analogy where there are similarities not identities, likenesses not equivalences, it would be begging the question to assert that such and such are like, but not identical, similar but not equivalent, analogous but not isomorphic.

*Student:* But this is precisely the argument why aleph facts alone will do. The *Case of Thorns* looks like the list of alpha facts from which we can induce a rule. However, it is a sort of index of type situations: the staff could have been a rifle butt or a mace or a club; the reeds could have been effluent of a chemical nature; a dead duck and so on. Then the reason given for the decision is not a causal but a logical account of what holds between the examples given by the judge.

*Metaphysician:* It is tempting to say that we already know what is material or relevant about the alpha facts which enables the judge to put the case as he did.

*Wrangler:* Well, one is tempted to say that the rule is prior to the right or the remedy, not merely temporally, but logically prior. One is tempted to say that the rule, whether it is of logic, mathematics or law is prior to the cases, and that what we do is to subsume the cases under the rule.

*Metaphysician:* But this is heresy—worse, it is a muddle. After all the Student argued in logic and the law in order to shew how

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<sup>16</sup> W. COOK, THE LOGICAL AND LEGAL BASES OF THE CONFLICT OF LAWS 29 (1942).

formalised systems, with their rules, laws and so on, also depended on the cases actual and possible.

*Student:* All we have to do is to leave the argument for a minute or two, bring up some other argument, as here when the inductive argument is brought up, for us to fall back on the old ideas. Because the law shares so many features with mathematics and logic, the Physicist asserts that it must be deductive if it is not inductive. Then because the law is shewn not to be deductive, it is said it must be inductive. It is almost tempting to say that the law is just law, that legal statements are just legal statements, that paradduction is just paradduction.

*Wrangler:* But if the rules depend upon the cases and you know which cases are which, how do you know which rules are which?

*Physicist:* If you know which cases are which, why is it not the case that the cases depend upon the rules? Why should the words and "the reasons for all these cases is . . ." be construed as the Student has done? Why should not the reason precede the cases?

*Student:* If it is asserted that the rule precedes the cases how does the argument proving this proposition proceed? Do we appeal to a general law or universal rule? If so, what is this rule? If we do not appeal to a general rule, which says in terms that all rules precede cases, then to what do we appeal? I suppose to rules which illustrate that *in Rule 1* the rule precedes the cases, and that *in Rule n* the rule precedes the cases. Now what sort of argument is this? Are we to run serially through all rules and thus shew that the rule is that rules always precede cases. This is an inductive argument which cannot prove a logical point. Perhaps the matter can be proved deductively but then we have to establish the premises or axioms, and this is the question at issue.

*Metaphysician:* The way to proceed from this point is to find out what is the nature of the question. If it is a philosophical matter I suggest that we can proceed by way of the case by case procedure. We are not here concerned with a causal question but with a logical enquiry. Induction will not do. We may try to prove the general rule by shewing that all cases of rule 1 precede the cases, and all cases of rule 2 precede the cases, that all cases of rule 3 precede the cases, for all cases from rule 1 to rule n. This proof however would not be the application of the rule but the case by case procedure. Thus we could establish the rule that all rules

precede the cases *only from cases of rules preceding cases themselves*. This is paradiuction par excellence.

*Student*: And this is analogous to claims logicians make about how the rules of inference are derived from formal analogy. It is analogous to arguments mathematicians use when they use mathematical induction.

*Wrangler*: I suppose that you argue that this conclusion can be established informally in the law.

*Student*: Yes.

*Wrangler*: With reference to *Shelley's Case* and other cases which establish rules, cases like *Wild's Case*,<sup>17</sup> *Whitby v. Mitchell*,<sup>18</sup> *Spencer's Case*,<sup>19</sup> *Tulk v. Moxhay*,<sup>20</sup> *Royal Bank v. Turquand*,<sup>21</sup> *Purefoy v. Rogers*,<sup>22</sup> *Sibley v. Perry*,<sup>23</sup> *Andrews v. Partington*,<sup>24</sup> *Pinnel's Case*<sup>25</sup> and so on. There are many of them.

*Student*: Quite so. There are lines of authorities which can be compared and contrasted. Denning J. did this in the *High Trees Case*,<sup>26</sup> where he noticed that the cases upon equitable estoppel resembled cases of contract unsupported by consideration.

*Wrangler*: Yes. What does this do to the axiom-theorem claim? Suppose that the law was so developed that all cases yielded rules so that the legal calculus was a calculus of rules, would there be any room for paradiuction?

*Doctor*: Indeed in civil law countries that is what happens when we have a code.

*Student*: Yet there are cases cited here as illustrations of the application and operation of the code. These are persuasive precedents.

*Doctor*: Yes, and Leibnitz suggested that there is a reason behind all this, and a reason worth the notice of logicians.

*Student*: This does not seem fatal. Theorems may be derived more formally by deduction from axioms or rules. New occasions and novel situations will still call for some moves as imaginative as

<sup>17</sup> 77 Eng. Rep. 277 (K.B. 1599).

<sup>18</sup> 42 Ch. D. 494 (1889), *appeal dismissed*, 44 Ch. D. 85 (1890).

<sup>19</sup> 77 Eng. Rep. 72 (K.B. 1583).

<sup>20</sup> 41 Eng. Rep. 1143 (Ch. 1848).

<sup>21</sup> 119 Eng. Rep. 474 (Q.B. 1855), *aff'd*, 119 Eng. Rep. 886 (Ex. 1856).

<sup>22</sup> 85 Eng. Rep. 1181 (K.B. 1670).

<sup>23</sup> 32 Eng. Rep. 211 (Ch. 1802).

<sup>24</sup> 29 Eng. Rep. 610 (Ch. 1791).

<sup>25</sup> 77 Eng. Rep. 237 (K.B. 1600).

<sup>26</sup> *Central London Property Trust, Ltd. v. High Trees House, Ltd.*, [1947] K.B. 130 (1946).



Gauss's or Einstein's move in comparing and contrasting on the one hand the series of natural numbers and the series of imaginary numbers by placing both in a Cartesian coordinate, or in comparing the theorems of relativity physics with those of Newtonian mechanics and Maxwell's field equations. The operation may not be the daily practice of the courts but that would not affect the logic we are considering.

*Wrangler:* You mean that when deduction fails us, paradiuction will be the last resort of the judges?

*Student:* Indeed. In the rule-ridden situation described, the lawyer would behave more and more like the mathematician and the symbolic logician. Formalisation and isomorphism would play a greater part in the legal game than cases and informal analogies, but this would not disturb the logical point which Wang made—that there is in isomorphism an application of analogy.

