

# **The Social Value of Mortality Risk Reduction: VSL vs. the Social Welfare Function Approach**

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**Abstract:** We examine how different welfarist frameworks evaluate the social value of mortality risk reduction. These frameworks include classical, distributively unweighted cost-benefit analysis—i.e., the “value per statistical life” (VSL) approach—and various social welfare functions (SWFs). The SWFs are either utilitarian or prioritarian, applied to policy choice under risk in either an “ex post” or “ex ante” manner. We examine the conditions on individual utility and on the SWF under which these frameworks display sensitivity to wealth and to baseline risk. Moreover, we discuss whether these frameworks satisfy related properties that have received some attention in the literature, namely equal value of risk reduction, preference for risk equity, and catastrophe aversion. We show that the particular manner in which VSL ranks risk-reduction measures is not necessarily shared by other welfarist frameworks.

**JEL:** D81, D61, D63, Q51.

**Keywords:** Value of statistical life, social welfare functions, cost-benefit analysis, equity, fairness, welfarism, risk policy.

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## Introduction

Two, quite different intellectual traditions exist concerning cost-benefit analysis (CBA). One view, dominant in the United States, sees CBA as a way to identify projects that pass a Kaldor-Hicks compensation test and advocates summing unweighted compensating or equivalent variations. Another approach, influential in the U.K. and Europe, sees the “social welfare function” (SWF) as the fundamental basis for policymaking.<sup>1</sup> CBA can generally mimic the effect of a SWF if compensating or equivalent variations are multiplied by distributive weights that reflect the declining marginal utility of wealth and also, perhaps, social inequality aversion (Adler 2012, pp. 109-10; Drèze and Stern 1987).

Scholarship regarding the “value per statistical life” (VSL) has generally taken the Kaldor-Hicks approach. VSL is the marginal rate of substitution between fatality risk in a specified time period, and wealth. In other words, it is the change in an individual’s wealth required to compensate him for a small change in his risk of dying during the period, divided by the risk change.<sup>2</sup>

The social value of mortality risk reduction presumably depends on our moral assumptions about risk and equity. American law (Executive Orders 12866 and 13563) instructs regulatory agencies to be sensitive to equity. The Institute of Medicine (2006) recommends that “The regulatory decision-making process should explicitly address and incorporate the distributional, ethical, and other implications of a proposed intervention along with the quantified results of [benefit-cost analysis] and [cost-effectiveness analysis].” Yet VSL has properties that can yield what are often viewed as inequitable evaluations of policy change. In particular, VSL does not value reductions in mortality risk equally. In some dimensions it favors those who are better off (e.g., individuals with higher wealth). In other dimensions, it favors the less well-off (e.g., individuals at higher risk of dying). But how does VSL compare with other frameworks?

This article examines the social value of mortality risk-reduction through the lens of a SWF. It asks: to what extent are the properties of VSL characteristic of various welfarist frameworks? If one views some of the implications of using VSL to value risk policies as inequitable, is there an SWF that exhibits a more attractive set of implications? In short, what happens if we shift from orthodox (distributively unweighted) CBA to some alternative SWF<sup>3</sup> as the societal tool for evaluating risk reductions?

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<sup>1</sup>To be sure, the concept of the SWF is hardly absent from scholarly discourse in the U.S. For example, it plays a central role in scholarship regarding optimal taxation (see, e.g., Kaplow 2008). However, it has been largely absent from U.S. scholarship and governmental practice regarding CBA.

<sup>2</sup>In the United Kingdom, VSL is often described as the “value per prevented fatality” (VPF) and interpreted as population aggregate willingness to pay to prevent a statistical fatality, which may depend on the nature of the tax system used to fund the risk reduction (Jones-Lee 1989; Baker et al. 2008).

<sup>3</sup>Note that CBA is locally equivalent to weighted utilitarianism with weights inversely proportional to individual marginal utility of wealth (as illustrated in Part II.A).

For choice under certainty, two SWFs are especially widespread in the literature: a utilitarian SWF, which sums individual utilities, and a prioritarian SWF, which sums a strictly concave function of individual utilities. Each of these can be applied under uncertainty in a variety of manners, yielding five functional forms that will be compared with CBA: plain utilitarianism (which we abbreviate as  $W^U$ ), ex post transformed utilitarianism ( $W^{EPTU}$ ), ex ante prioritarianism ( $W^{EAP}$ ), ex post untransformed prioritarianism ( $W^{EPUP}$ ), and ex post transformed prioritarianism ( $W^{EPTP}$ ).

Part I reviews the SWF approach and describes the five SWFs just mentioned. Part II presents a simple model of policy evaluation, and uses it to define the concept of the “social value of risk reduction”: the marginal social value of change in an individual’s survival probability, that is,  $\partial W / \partial p_i$ , with  $p_i$  individual  $i$ ’s survival probability. This concept can also be defined for CBA: in this case, the social value of risk reduction is just  $VSL_i$ .

In the remainder of the Article, we characterize the social value of risk reduction for the five SWFs and CBA, focusing in particular on the properties of “wealth sensitivity” and “sensitivity to baseline risk.”

*Wealth sensitivity:* Does the social value of risk reduction increase with individual wealth? As is well known, VSL increases with wealth but cross-sectional differences in VSL attributable to wealth are almost always suppressed in policy evaluation. Public /political resistance to differentiating VSL by wealth is so strong that use of a different (higher) VSL was rejected in a context where both the costs and benefits of regulation would fall on an identified higher-income group (airline passengers; Viscusi 2009). In contrast, increases in VSL attributable to future income growth are often incorporated in analyses (Robinson 2007). As we shall see, the social value of risk reduction increases with wealth for CBA and for the utilitarian SWFs,  $W^U$  and  $W^{EPTU}$ . By contrast, the prioritarian SWFs ( $W^{EAP}$ ,  $W^{EPUP}$ , and  $W^{EPTP}$ ) need not be positively sensitive to individual wealth in valuing risk reduction.

*Sensitivity to baseline risk.* Does the social value of risk reduction depend on the individual’s baseline risk of dying? This property, the “dead-anyway effect” (Pratt and Zeckhauser 1996) is not only of intrinsic interest, but is closely connected to the problem of statistical versus identified lives (Hammit and Treich 2007) and to the “rule of rescue,” a moral imperative for decision makers to give priority to people at higher risk (Jonsen 1986). Sensitivity to baseline risk is also closely related to the property of “risk equity”: preferring a policy that equalizes individuals’ risks of dying, as discussed by Keeney (1980) and reflected in concerns for environmental justice (Lazarus 1993). We note however that it has been recommended in some policy circles to not adjust the value of lifesaving programs for the health status of the affected population (European Commission 2001; Neumann and Weinstein 2010). As we shall demonstrate, the social value of risk reduction increases with baseline risk for CBA and for  $W^{EAP}$ , and does so for  $W^{EPTU}$  and  $W^{EPTP}$  under certain parameter assumptions, but is independent of baseline risk for  $W^U$  and  $W^{EPUP}$ .

Note that a social evaluation methodology that is *either* wealth-sensitive *or* sensitive to baseline risk *cannot* have the property of “equal value of risk reduction,” such that  $\partial W / \partial p_i$  is identical for all individuals. The nearly ubiquitous use of a single VSL by each governmental agency, the pressure to standardize VSLs among agencies (e.g., HM Treasury 2011) or among countries (see, e.g., Fankhauser et al. 1997, in the context of climate change), the proscription of an evaluation measure “that discounts the value of a life because of an individual’s disability” (U.S. Patient Protection and Affordable Care Act, quoted in Neumann and Weinstein 2010), and the adverse reaction to using a different (smaller) VSL for older people in EPA air regulations (Viscusi 2009) are consistent with widespread interest in equal value of risk reduction. However, equal value of risk reduction is not satisfied by CBA or by any of the SWFs except for  $W^{EPUP}$  and  $W^{EPTP}$  under restrictive parameter assumptions.

In the final part of the article, we turn to the property of *catastrophe aversion*. If a policy does not change the expected number of deaths, but reduces the chance of multiple individuals dying, does that count as a social improvement? It is widely noted that incidents in which many people die (e.g., an airliner crash or a nuclear disaster) are regarded as worse than an equal number of fatalities in unrelated events (e.g., traffic crashes or heart attacks) and catastrophic potential appears to be a major determinant of risk perceptions (Slovic 2000). However, empirical evidence suggests that the public does not support using a larger VSL for catastrophic risks (Jones-Lee and Loomes 1995; Covey et al. 2010; Rheinberger 2010). Keeney (1980) shows that a preference for risk equity (defined as greater similarity among individual risks) is incompatible with catastrophe aversion. While CBA,  $W^U$ ,  $W^{EAP}$ , and  $W^{EPUP}$  do not satisfy catastrophe aversion,  $W^{EPTU}$  and  $W^{EPTP}$  will do so with a concave social transformation function.

Our analysis puts CBA using VSL in a new light. CBA *is* wealth sensitive and *is* sensitive to baseline risk, but there are SWFs that lack one or both properties; conversely, CBA is not catastrophe averse, but some SWFs are. In short, we demonstrate that the particular manner in which VSL ranks risk-reduction measures is not the *inevitable* result of a welfarist approach to policymaking. VSL’s salient features can, if seen as undesirable, be mitigated by shifting to some alternative social welfare function.

Short proofs are provided in the text or footnotes, with longer proofs relegated to an appendix.

## I. SWFs Under Uncertainty

The SWF approach assumes some interpersonally comparable vector-valued utility function  $\mathbf{u}(\cdot)$ . If  $x$  is an outcome, then  $\mathbf{u}(x) = (u_1(x), \dots, u_N(x))$  where  $u_i(x)$  is a real number, with  $N$  individuals in the population.<sup>4</sup> (Throughout this article, we assume that  $N$  is the same in all

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<sup>4</sup> The  $i$ th argument of  $\mathbf{u}(x)$ , denoted  $u_i(x)$ , represents the well-being level of individual  $i$  in outcome  $x$ . Function  $\mathbf{u}(\cdot)$  is “interpersonally comparable” in the sense that these numbers represent how well-being levels and differences are

outcomes.) An SWF is a rule  $R$  for ranking outcomes as a function of their associated utility vectors. It says:  $x \succcurlyeq y$  iff  $\mathbf{u}(x) R \mathbf{u}(y)$  (where “iff” means “if and only if”). The literature discusses standard forms for  $R$ . One is a utilitarian SWF:  $x \succcurlyeq y$  iff  $\sum_{i=1}^N u_i(x) \geq \sum_{i=1}^N u_i(y)$ . Another is a “prioritarian” (additively separable, concave) SWF:  $x \succcurlyeq y$  iff  $\sum_{i=1}^N g(u_i(x)) \geq \sum_{i=1}^N g(u_i(y))$ , with  $g(\cdot)$  a strictly increasing and concave real-valued function. A third is the “leximin” SWF, which ranks utility vectors according to their smallest entries, if these are equal their second-smallest, etc. The “rank-weighted” SWF uses fixed weights  $\alpha_1 > \alpha_2 \dots > \alpha_N$ , with  $\alpha_1$  the weight for the smallest utility in a vector,  $\alpha_2$  the second smallest, etc., and ranks vectors by summing weighted utilities.<sup>5</sup> (On the different functional forms for an SWF, see generally Adler 2012; Bossert and Weymark 2004; Blackorby, Bossert and Donaldson 2005.)

As recent scholarship has shown, a wide range of possibilities exist for applying an SWF under uncertainty, with different axiomatic characteristics. (See generally Fleurbaey 2010; see also Adler 2012, Chapter 7.) In representing policy choice under uncertainty, we will use a standard Savage-style model where there is a set of states and a fixed probability assigned to each state  $s$ ,  $\pi_s$ . An action (e.g., governmental policy) maps each state onto an outcome. Let  $x^{a,s}$  be the outcome of action  $a$  in state  $s$ .

Consider first the possibilities for a utilitarian SWF. “Ex post” *untransformed* utilitarianism assigns each action a number equaling the expected value of the sum of individual utilities. In other words,  $a \succcurlyeq b$  iff  $W(a) \geq W(b)$ , with  $W(a) = \sum_s \pi_s \sum_i u_i(x^{a,s})$ . “Ex post” untransformed utilitarianism yields the same ranking of actions as “ex ante” utilitarianism, ranking actions according to the sum of individual expected utilities. Let  $U_i(a) = \sum_s \pi_s u_i(x^{a,s})$ . Then “ex ante” utilitarianism says:  $a \succcurlyeq b$  iff  $W(a) \geq W(b)$ , with  $W(a) = \sum_i U_i(a)$ .

Ex post utilitarianism can also take a “transformed” form. Let  $h(\cdot)$  be a strictly increasing (but not necessarily linear) function. Then ex post *transformed* utilitarianism sets  $W(a) = \sum_s \pi_s h(\sum_i u_i(x^{a,s}))$ . Note that, if  $h(\cdot)$  is non-linear, ex post transformed utilitarianism need not rank actions the same way as ex ante utilitarianism.

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compared between persons. For example,  $u_i(x) > u_j(y)$  iff individual  $i$  in outcome  $x$  is better off than individual  $j$  in outcome  $y$  (from the perspective of a social decision maker). On interpersonal comparability, see generally Adler (2012, Chapters 2 and 3).

<sup>5</sup> Let  $\tilde{u}_1(x) \leq \tilde{u}_2(x) \leq \dots \leq \tilde{u}_N(x)$  denote a rank-ordered permutation of the vector  $\mathbf{u}(x)$ . Then the rank-ordered SWF ranks outcomes as follows, using some fixed set of strictly decreasing weights  $\alpha_1, \dots, \alpha_N$ :  $x \succcurlyeq y$  iff  $\sum_i \alpha_i \tilde{u}_i(x) \geq \sum_i \alpha_i \tilde{u}_i(y)$ , with  $x$  and  $y$  two outcomes.

Consider, next, the possibilities for a prioritarian SWF. “Ex post” *untransformed* prioritarianism assigns each action a number equaling the expected value of the sum of a strictly increasing and concave function of individual utility. In other words,  $a \succcurlyeq b$  iff  $W(a) \geq W(b)$ , with  $W(a) = \sum_s \pi_s \sum_i g(u_i(x^{a,s}))$ . While ex post untransformed utilitarianism is mathematically equivalent to ex ante utilitarianism, ex post untransformed prioritarianism is not equivalent to ex ante prioritarianism, where  $W(a) = \sum_i g(U_i(a))$ . Finally, ex post *transformed* prioritarianism should be mentioned:  $W(a) = \sum_s \pi_s h\left(\sum_i g(u_i(x^{a,s}))\right)$ , with  $h(\cdot)$  strictly increasing.

Fleurbaey (2010) focuses on the properties of a particular kind of ex post transformation: the “equally distributed equivalent” (EDE). Let  $w(\cdot)$  be a function from utility vectors to real numbers corresponding to a particular SWF.<sup>6</sup> Let  $u^*$  be such that  $w(u^*, u^*, \dots, u^*) = w(\mathbf{u}(x))$  for a given outcome  $x$ . Define the real-valued function  $h^{EDE}(\cdot)$  as follows:  $h^{EDE}(w(\mathbf{u}(x))) = u^*$ . In the case of the utilitarian SWF,  $h^{EDE}(\cdot)$  is just average utility:  $h^{EDE}\left(\sum_i u_i(x)\right) = (1/N) \sum_i u_i(x)$ , i.e.,  $h^{EDE}(w) = w/N$ . In this case,  $h^{EDE}(\cdot)$  is a linear function. By contrast, in the case of the prioritarian SWF,  $h^{EDE}(\cdot)$  is strictly *convex*. Note that

$$h^{EDE}\left[\sum_i g(u_i(x))\right] = g^{-1}\left[(1/N) \sum_i g(u_i(x))\right], \text{ i.e., } h^{EDE}(w) = g^{-1}(w/N), \text{ leading to } W(a) = \sum_s \pi_s g^{-1}\left(1/N \sum_i g(u_i(x^{a,s}))\right) \text{ in the case of ex post prioritarianism.}$$

For simplicity, we will not consider the rank-weighted SWF or the leximin SWF. Instead, our focus will be on different possible methodologies for applying a utilitarian or prioritarian SWF to value risk-reduction measures.<sup>7</sup>

## II. VSL versus SWF: A Simple Model

For the remainder of the article, unless otherwise noted, we use “CBA” to mean cost-benefit analysis without distributive weights. CBA ranks policies by summing equivalent or compensating variations. As is well known, CBA does not provide a social ranking—it can

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<sup>6</sup> In the case of the utilitarian SWF,  $w(\mathbf{u}(x)) = \sum_i u_i(x)$ ; in the case of the prioritarian SWF,  $w(\mathbf{u}(x)) = \sum_i g(u_i(x))$ ; and for the rank-weighted SWF,  $w(\mathbf{u}(x)) = \sum_i \alpha_i \tilde{u}_i(x)$ .

<sup>7</sup> Some authors, e.g., Ben Porath, Gilboa and Schmeidler (1997), Ulph (1982), have characterized a “hybrid” approach. Let  $W(a)$  be the value assigned to an action by ex post (transformed or untransformed) utilitarianism, prioritarianism, or the rank-weighted approach, and  $W^*(a)$  the value assigned by, respectively, ex ante utilitarianism, prioritarianism, or the rank weighted approach. Then if  $\lambda$  is between 0 and 1, the hybrid approach assigns each action a value equaling  $\lambda W(a) + (1 - \lambda) W^*(a)$ . This approach, too, is beyond the scope of the current article.

violate completeness and transitivity (Blackorby and Donaldson 1986). However, we can use CBA to define a social ranking of alternatives using equivalent variations from a fixed baseline.<sup>8</sup> Consider some baseline action  $O$ , the “status quo” action. Let  $a, b, \dots$  be other possible actions (governmental policies). For a given such action  $a$ , let individual  $i$ ’s equivalent variation  $EV_i^a$  be the change to individual  $i$ ’s wealth in every state of the world, in  $O$ , sufficient to make  $i$  ex ante indifferent as between  $O$  and  $a$ . Then we will say that CBA ranks actions by saying:  $a \succcurlyeq b$  iff  $W^{CBA}(a) \geq W^{CBA}(b)$ , where  $W^{CBA}(a) = \sum_i EV_i^a$ .

In order to compare CBA to various SWFs, we adopt the following simple, one-period model—one that is frequently used in the discussion of VSL. Each policy  $a, b, \dots$  is such that each individual has the same wealth ( $c_i^a, c_i^b$ , etc.) in all states as a result of that policy (although *not* necessarily the same across policies or individuals.). Thus the model allows for interpersonal differences in wealth, and for a policy to cause changes in an individual’s wealth (although we will not focus on policy-induced changes in wealth in this article).

For a given policy, the state determines which individuals will be alive or dead. We introduce  $l_i^{a,s}$ , which takes the value 1 if individual  $i$  is alive and 0 if dead. Utility functions  $u(\cdot)$  and  $v(\cdot)$  are the (common and interpersonally comparable) utility functions of wealth if individuals are alive and dead, respectively (i.e.,  $v(\cdot)$  is the bequest function).

We assume, as is standard in the VSL literature, that  $u(c) > v(c)$ ,  $u'(c) > v'(c) \geq 0$ , and  $u''(c) \leq 0$ ,  $v''(c) \leq 0$ . We refer to this set of assumptions as the “standard” utility model (although it should be recognized that the assumptions are not *entailed* by expected utility theory; we relax some of them in Part V.B).

Let  $p_i^a$  be individual  $i$ ’s probability of being alive with policy  $a$ , that is,  $p_i^a = \sum_s \pi_s l_i^{a,s}$ .

Then  $U_i(a)$ , individual  $i$ ’s expected utility with action  $a$ , is simply  $p_i^a u(c_i^a) + (1 - p_i^a) v(c_i^a)$ .

Some of our results depend upon a zero bequest function, i.e.,  $v(c) = 0$  for all  $c$ . Note that this is consistent with the standard utility model.<sup>9</sup>

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<sup>8</sup> Alternatively, one can construct a ranking using the sum of compensating variations from a fixed baseline where individual  $i$ ’s compensating variation  $CV_i^a$  is the change to individual  $i$ ’s wealth in every state of the world, in  $a$ , sufficient to make  $i$  ex ante indifferent as between  $O$  and  $a$ ,  $CV_i^b$  is the analogous change to individual  $i$ ’s wealth in every state of the world, in  $b$ , and so forth. The social ranking based on compensating variation can violate the Pareto principle, while the social ranking based on equivalent variation cannot. The reason is that the individual’s marginal utility of wealth can depend on the state of the world (e.g., if he lives or dies). An individual may prefer  $a$  to  $b$ , but if his marginal utility of wealth in  $a$  exceeds his marginal utility of wealth in  $b$ ,  $CV_i^a$  can be smaller than  $CV_i^b$ . If no one else in the population is affected by shifting from the status quo to  $a$  or to  $b$ , then  $a$  is Pareto superior to  $b$  yet CBA using compensating variation will rank  $b$  superior to  $a$ . This situation cannot arise using the social ranking based on equivalent variation from a fixed baseline, which always adds wealth to the states associated with the same action (the status quo action  $O$ ).

<sup>9</sup> In distinguishing between the case where  $v(c) = 0$  and  $v(c) \neq 0$ , we are assuming that the common, interpersonally comparable utility function  $u^*(c, l)$  that gives rise to  $u(\cdot)$  and  $v(\cdot)$ — $l$  a variable indicating whether the individual is

A. The Social Value of Risk Reduction: The Benchmark Case

We first use this simple model to define the social value of risk reduction for three “benchmark” SWFs: ex post untransformed utilitarianism (which is equivalent, recall, to ex ante utilitarianism); ex post untransformed prioritarianism, and ex ante prioritarianism. As a shorthand, we will refer to ex post untransformed utilitarianism/ex ante utilitarianism as the “plain utilitarian” SWF—by contrast with the ex post *transformed* utilitarian SWF. Each of these approaches (like CBA or other SWFs) ranks policies via a rule of the form:  $a \succcurlyeq b$  iff  $W(a) \geq W(b)$ . Moreover, in the case of the simple one-period model under discussion, the  $W$ -functions associated with the three benchmark SWFs are especially tractable.

Let  $W^U$ ,  $W^{EPUP}$ , and  $W^{EAP}$  denote the  $W$ -functions associated, respectively, with plain utilitarianism, ex post untransformed prioritarianism, and ex ante prioritarianism. Then the following can be straightforwardly established:

$$W^U(a) = \sum_i U_i(a)$$

$$W^{EPUP}(a) = \sum_i \left[ p_i^a g(u(c_i^a)) + (1 - p_i^a) g(v(c_i^a)) \right]$$

$$W^{EAP}(a) = \sum_i g(U_i(a)).$$

Note that, in each case—as with  $W^{CBA}$ —the ranking of policies is just a function of each individual’s wealth and the probability of her death. Moreover, these simple formulas (as with  $W^{CBA}$ ) hold true regardless of the degree to which individuals’ survival risks are correlated. They hold true *both* in the case of statistically independent survival risks *and* in the case where there are some pairs of individuals whose survival risks are positively or negatively correlated.

Furthermore, in the case where policies represent a small variation in individual risk and/or wealth around the status quo policy  $O$ , we can use the total differential to approximate a change in  $W^{CBA}$ ,  $W^U$ ,  $W^{EPUP}$ , and  $W^{EAP}$ . As a shorthand, and without risk of confusion, we will use the term  $p_i$  to mean individual  $i$ ’s survival probability in the status quo (strictly,  $p_i^O$ );  $c_i$  to mean individual  $i$ ’s wealth in the status quo (strictly,  $c_i^O$ );  $U_i$  her expected utility in the status quo (strictly,  $U_i^O$ ); and a function incorporating these terms (such as  $\partial U_i / \partial p_i$ ) to mean the function evaluated at the status quo values (here,  $\partial U_i / \partial p_i$  evaluated at the values  $U_i^O$  and  $p_i^O$ ).

Consider, now, some policy  $a$  that changes each individual  $i$ ’s survival probability by  $dp_i^a$  and her wealth by  $dc_i^a$ . Then it can be seen that:

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alive or dead—is unique up to a positive ratio transformation, not merely a positive affine transformation. Prioritarian SWFs, indeed, make stronger assumptions on the measurability of utility than utilitarianism.

$$dW^{CBA}(a) = \sum_i [dc_i^a + VSL_i dp_i^a],$$

where  $VSL_i$  is individual  $i$ 's marginal rate of substitution between survival probability and wealth in  $O$ , i.e.,  $\frac{\partial U_i / \partial p_i}{\partial U_i / \partial c_i}$ , which equals  $\frac{u(c_i) - v(c_i)}{p_i u'(c_i) + (1 - p_i) v'(c_i)}$ .<sup>10</sup> Similarly,

$$dW^U(a) = \sum_i [dc_i^a [p_i u'(c_i) + (1 - p_i) v'(c_i)] + dp_i^a [u(c_i) - v(c_i)]]$$

$$dW^{EPUP}(a) = \sum_i [dc_i^a [p_i g'(u(c_i)) u'(c_i) + (1 - p_i) g'(v(c_i)) v'(c_i)] + dp_i^a [g(u(c_i)) - g(v(c_i))]]$$

$$dW^{EAP}(a) = \sum_i [dc_i^a [g'(U_i)(p_i u'(c_i) + (1 - p_i) v'(c_i))] + dp_i^a [g'(U_i)(u(c_i) - v(c_i))]].$$

It is useful to think of  $W^{CBA}$ ,  $W^U$ ,  $W^{EPUP}$ , and  $W^{EAP}$  as different methodologies for assigning a “social value” to policies. Note that, in each case, the total differential allows us to distinguish (1) the change in “social value” associated with the change in individual  $i$ 's wealth ( $dc_i$ ) from (2) the change in “social value” associated with the change in her survival probability ( $dp_i$ ). The latter change is just  $(\partial W / \partial p_i) dp_i$ . For short, let us say that the social value of risk reduction, for a given individual  $i$ , according to a given  $W$ , is just  $\partial W / \partial p_i$ . (To be clear, this social value may well depend upon  $i$ 's wealth in the status quo  $c_i$ , her survival probability  $p_i$ , or both.)

The social values of risk reduction, for CBA and the three benchmark SWFs, are as follows:

$$\frac{\partial W^{CBA}}{\partial p_i} = VSL_i = \frac{u(c_i) - v(c_i)}{p_i u'(c_i) + (1 - p_i) v'(c_i)}$$

$$\frac{\partial W^U}{\partial p_i} = u(c_i) - v(c_i)$$

$$\frac{\partial W^{EPUP}}{\partial p_i} = g(u(c_i)) - g(v(c_i))$$

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<sup>10</sup> Note that  $dW^{CBA}$  can be obtained from  $dW^U$  by weighting  $dc_i$  and  $dp_i$  by the inverse of the expected marginal utility of wealth,  $p_i u' + (1 - p_i) v'$ . In other words, CBA is locally equivalent to weighted utilitarianism with weights inversely proportional to marginal utility of wealth.

$$\frac{\partial W^{EAP}}{\partial p_i} = g'(U_i)(u(c_i) - v(c_i)).$$

Because CBA and each of the three benchmark social welfare functions are additively separable across individuals, the social value of a policy that changes several individuals' risks is simply the sum of the social values of the individual changes.

#### B. Ex Post Transformed Utilitarianism and Prioritarianism

Even using the simple model set forth in this part, the social value that the ex post transformed utilitarian and prioritarian SWFs assign to the status quo or a given policy *cannot* be expressed as a function of individuals' wealth amounts and survival probabilities without considering the extent to which individual risks are correlated. Thus, with these two SWFs, it is not meaningful to speak of the social value of reducing a given individual's risk, as a function of her wealth and survival probability, without further information about the correlation of her survival with others'.

To illustrate, consider the ex post transformed utilitarian SWF with  $h(\cdot)$  the logarithm. Assume that  $u(c) = \sqrt{c}$ ,  $v(c) = \sqrt{c}/2$ , and some state  $s^*$  has probability 0.25. There are three individuals, all with wealth 100: Joe, Jane, and Sally. Imagine, first, that in the status quo Joe survives in other states and dies in  $s^*$ ; Jane and Sally also die in  $s^*$ ; and a policy saves Joe in  $s^*$ , reducing his fatality risk from 0.25 to 0. The social value of this individual risk reduction is  $0.25 \cdot (\ln(20) - \ln(15))$ . Imagine, now, that in the status quo Jane and Sally survive in  $s^*$ , and the policy once more saves Joe in  $s^*$ , again reducing his fatality risk from 0.25 to 0. Now, the social value of this risk reduction is  $0.25 \cdot (\ln(30) - \ln(25))$ .

The general framework for transformed settings can be introduced as follows. Let  $X_k = (u(c_k), v(c_k); p_k)$ ,  $k=1, \dots, N$ , be a random variable which indicates that  $X_k$  equals  $u(c_k)$  with probability  $p_k$  and equals  $v(c_k)$  otherwise. Welfare under ex post transformed utilitarianism and under ex post transformed prioritarianism are then given respectively by:

$$W^{EPTU} = Eh\left(\sum_{k=1}^N X_k\right)$$

$$W^{EPTP} = Eh\left(\sum_{k=1}^N g(X_k)\right).$$

Consistent with the example above, observe that the exact relationship between either  $W^{EPTU}$  or  $W^{EPTP}$  and survival probabilities depends on the correlations across the  $X_k$   $k=1, \dots, N$ , expressed through the expectation operator  $E$ . It is beyond the scope of this article to provide a full treatment of the social value of risk reduction for the ex post transformed SWFs. Rather, we

consider the case of independent individual risks, which correspond to the case where the random variables  $X_k, k=1, \dots, N$ , are statistically independent.

As with CBA and the benchmark SWFs, we can use the total differential to obtain:

$$dW^{EPTU}(a) = \sum_i \frac{\partial W^{EPTU}}{\partial c_i} dc_i + \sum_i \frac{\partial W^{EPTU}}{\partial p_i} dp_i \text{ and } dW^{EPTP}(a) = \sum_i \frac{\partial W^{EPTP}}{\partial c_i} dc_i + \sum_i \frac{\partial W^{EPTP}}{\partial p_i} dp_i ,$$

with the derivatives evaluated at status quo wealth and survival probability. The social value of risk reduction for individual  $i$  is  $\frac{\partial W^{EPTU}}{\partial p_i}$  for the ex post transformed utilitarian SWF, and

$$\frac{\partial W^{EPTP}}{\partial p_i} \text{ for the ex post transformed prioritarian SWF.}$$

Under the assumption of statistical independence, we can derive closed-form expressions for these social values. To do so, it is useful to define the indirect function

$$H(x) = Eh(x + \sum_{k \neq i, j} X_k). \text{ Observe that } H(x) \text{ inherits the properties of } h(x); \text{ in particular } H(x) \text{ is}$$

increasing iff  $h(x)$  is increasing, and  $H(x)$  is concave (convex) iff  $h(x)$  is concave (convex).

Then we can write

$$W^{EPTU} = p_i p_j H(u(c_i) + u(c_j)) + p_i (1 - p_j) H(u(c_i) + v(c_j)) + (1 - p_i) p_j H(v(c_i) + u(c_j)) + (1 - p_i) (1 - p_j) H(v(c_i) + v(c_j)).$$

We then obtain the social value of risk-reduction under ex post transformed utilitarianism, which is given by

$$\frac{\partial W^{EPTU}}{\partial p_i} = p_j H(u(c_i) + u(c_j)) + (1 - p_j) H(u(c_i) + v(c_j)) - p_j H(v(c_i) + u(c_j)) - (1 - p_j) H(v(c_i) + v(c_j)).$$

The social value of risk-reduction under ex post transformed prioritarianism can be obtained in a similar fashion (by simply replacing  $X_k$  and its realizations by  $g(X_k)$ ). These

closed-form expressions for  $\frac{\partial W^{EPTU}}{\partial p_i}$  and  $\frac{\partial W^{EPTP}}{\partial p_i}$  in terms of the survival probability of

individual  $i$ , the wealth and survival probability of some other individual  $j$ , and the indirect function  $H$  (which takes account of the wealth and survival probabilities of everyone else in the

population) are useful in examining the sensitivity to wealth and to baseline risk of  $\frac{\partial W^{EPTU}}{\partial p_i}$  and

$$\frac{\partial W^{EPTP}}{\partial p_i}.$$
 <sup>11</sup>

### III. Wealth Sensitivity

The prior part defined the social value of risk reduction,  $\partial W / \partial p_i$ , for CBA, three benchmark SWFs, and two transformed SWFs (assuming independent survival outcomes). We now ask: How do these different approaches *compare* in assigning social value to risk reduction? In the status quo, individual  $i$  has wealth  $c_i$  and survival probability  $p_i$ , while individual  $j$  has a different amount of wealth  $c_j$  and/or a different survival probability  $p_j$ . How does the social value of risk reduction for the first individual,  $\partial W / \partial p_i$ , compare with the social value of risk reduction for the second,  $\partial W / \partial p_j$ — with “social value” calculated using  $W^{CBA}$  or, alternatively,  $W^U$ ,  $W^{EPUP}$ ,  $W^{EAP}$ ,  $W^{EPTU}$ , or  $W^{EPTP}$ ?

Consider the case where individual  $i$  has more status quo wealth than individual  $j$  ( $c_i > c_j$ ) and both have the same survival probability  $p_i = p_j$ . This set-up allows us to isolate the effect of individual wealth on the social value of individual risk reduction. We define a social ranking as (positively) wealth sensitive if it always assigns higher value to reducing the risk of the wealthier of two individuals having the same mortality risk.

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<sup>11</sup> A different and perhaps slightly more transparent closed-form expression for these social values is as follows. Let  $\mathbf{N}$  be the set of individuals and  $\mathbf{M}$  a subset of  $\mathbf{N}$ . Let  $P_i(\mathbf{M})$  denote  $\prod_{k \in \mathbf{M}} p_k^o \prod_{l \in \mathbf{N} \setminus [\mathbf{M} \cup \{i\}]} (1 - p_l^o)$ . And let

$S_i(\mathbf{M})$  denote  $\sum_{k \in \mathbf{M}} u(c_k^o) + \sum_{l \in \mathbf{N} \setminus [\mathbf{M} \cup \{i\}]} v(c_l^o)$ . Then it can be shown that:

$$\frac{\partial W^{EPTU}}{\partial p_i} = \sum_{\mathbf{M} \subseteq [\mathbf{N} \setminus \{i\}]} P_i(\mathbf{M}) [h(S_i(\mathbf{M}) + u(c_i)) - h(S_i(\mathbf{M}) + v(c_i))].$$
 Similarly, let  $G_i(\mathbf{M})$  denote

$$\sum_{k \in \mathbf{M}} g(u(c_k)) + \sum_{l \in \mathbf{N} \setminus [\mathbf{M} \cup \{i\}]} g(v(c_l)).$$
 Then:  $\frac{\partial W^{EPTP}}{\partial p_i} = \sum_{\mathbf{M} \subseteq [\mathbf{N} \setminus \{i\}]} P_i(\mathbf{M}) [h(G_i(\mathbf{M}) + g(u(c_i))) - h(G_i(\mathbf{M}) + g(v(c_i)))]$ .

These formulas make it clear how  $\frac{\partial W^{EPTU}}{\partial p_i}$  and  $\frac{\partial W^{EPTP}}{\partial p_i}$  depend not only on individual  $i$ 's attributes, but

also upon the wealth and survival probabilities of everyone else in the population. By contrast, as can be seen from the analysis in Part II.A above, the social values of risk reduction for CBA and the three benchmark SWFs depend *only* upon the wealth  $c_i$  and survival probability  $p_i$  of the individual  $i$  whose risk is being reduced.

**Definition 1:** Let  $c_i > c_j$  and  $p_i = p_j$ . A social ranking is (positively) wealth sensitive iff

$$\frac{\partial W}{\partial p_i} > \frac{\partial W}{\partial p_j}.$$

Note that the social value of risk reduction is positive for all of the  $W$ -functions considered here, regardless of individual wealth and baseline risk. Thus  $\frac{\partial W}{\partial p_i} > \frac{\partial W}{\partial p_j}$  iff  $\frac{\partial W / \partial p_i}{\partial W / \partial p_j} > 1$ . In what follows, we often focus on the ratio  $\frac{\partial W / \partial p_i}{\partial W / \partial p_j}$ .<sup>12</sup>

We first discuss the wealth sensitivity of CBA and the three benchmark SWFs. CBA is (positively) sensitive to individual wealth. As is well known, CBA assigns the wealthier individual a greater social value of individual risk reduction:  $VSL_i / VSL_j > 1$ . The same is true of plain utilitarianism:  $\frac{u(c_i) - v(c_i)}{u(c_j) - v(c_j)} > 1$ , on the assumption that  $u'(\cdot) > v'(\cdot)$ .

However, ex post untransformed prioritarianism and ex ante prioritarianism do not necessarily assign the wealthier individual a greater social value of risk reduction. In the case of ex post untransformed prioritarianism, the relevant ratio is  $\frac{g(u(c_i)) - g(v(c_i))}{g(u(c_j)) - g(v(c_j))}$ . In the case of ex ante prioritarianism, it is  $\frac{g'(U_i)(u(c_i) - v(c_i))}{g'(U_j)(u(c_j) - v(c_j))}$ . With  $c_i > c_j$ , these ratios can be greater than, less than, or equal to one, depending on the functional forms of  $g(\cdot)$ ,  $u(\cdot)$ , and  $v(\cdot)$ .<sup>13</sup> Under prioritarianism, there is a tension between the positive effect of wealth on the individual's utility gain from survival and its negative effect on her social priority. We therefore arrive at our first result.

**PROPOSITION I: CBA and plain utilitarianism are (positively) wealth sensitive: the social value of individual risk reduction increases with individual wealth. In the case of ex post untransformed prioritarianism and ex ante prioritarianism, the social value of individual**

<sup>12</sup> In this article, we are interested in the *ordinal* properties of the different  $W$  functions ( $W^{CBA}$ ,  $W^U$ , etc.), i.e., the ordinal ranking of policies that they generate. Our interest in the ratio just described is consistent with the fact that  $W$  merely has ordinal significance. Let  $f(\cdot)$  be any differentiable, strictly increasing function. Then  $\frac{\partial f(W)}{\partial p_i} > \frac{\partial f(W)}{\partial p_j}$  iff

$$f'(W) \frac{\partial W}{\partial p_i} > f'(W) \frac{\partial W}{\partial p_j} \text{ iff } \frac{\partial W / \partial p_i}{\partial W / \partial p_j} > 1, \text{ since } f'(\cdot) > 0 \text{ as are } \partial W / \partial p_i \text{ and } \partial W / \partial p_j.$$

<sup>13</sup> Consider, first, ex post untransformed prioritarianism. The ratio is greater than one in the case of a zero bequest function, the case considered immediately below. Alternatively, let  $v(\cdot) = ku(\cdot)$  with  $0 < k < 1$ . With  $g(x) = \log x$ , the ratio is unity while with  $g(x) = -1/x$  for instance, the ratio is less than one. Next consider ex ante prioritarianism. As discussed immediately below, the ratio can be greater than, less than, or equal to one even if the bequest function is constrained to be zero, and *a fortiori* without such constraint.

**risk reduction can increase with individual wealth, decrease with individual wealth, or remain constant—depending on the functional forms of  $g(\cdot)$ ,  $u(\cdot)$ , and  $v(\cdot)$ .**

This is an important result. CBA's positive wealth sensitivity in valuing risk reduction does not emerge as a general feature of welfarism (even if we confine our attention to the three benchmark SWFs, let alone other SWFs). Although  $VSL_i$  increases with individual wealth, that is not necessarily true of  $\frac{\partial W^{EPUP}}{\partial p_i}$  or  $\frac{\partial W^{EAP}}{\partial p_i}$ .

Can we achieve clearer results regarding the wealth sensitivity of ex post untransformed prioritarianism and ex ante prioritarianism by restricting the bequest function to be zero ( $v(\cdot) = 0$ )? With a zero bequest function, ex post untransformed prioritarianism is only well-defined if  $g(0)$  is well-defined.<sup>14</sup> Continuing to focus on the case where  $c_i > c_j$  and  $p_i = p_j$ , the ratio  $\frac{\partial W^{EPUP} / \partial p_i}{\partial W^{EPUP} / \partial p_j}$  becomes  $\frac{g(u(c_i)) - g(0)}{g(u(c_j)) - g(0)}$ , which is greater than unity, since  $g(\cdot)$  and  $u(\cdot)$  are strictly increasing.

However, even with a zero bequest function, ex ante prioritarianism may be insensitive or negatively sensitive to wealth. Note that the ratio  $\frac{\partial W^{EAP} / \partial p_i}{\partial W^{EAP} / \partial p_j}$  becomes  $\frac{g'(U_i)u(c_i)}{g'(U_j)u(c_j)}$ . Setting  $g(\cdot) = \log$  makes this ratio unity. Moreover, if the  $g(\cdot)$  function is more concave than the logarithm, ex ante prioritarianism is *negatively* wealth-sensitive—assigning a *lower* social value to risk reduction for wealthier individuals.<sup>15</sup>

Let us turn now to the wealth-sensitivity properties of the *transformed* SWFs (with independent survival risks).<sup>16</sup> From the equality obtained in Part II, and assuming  $p_i = p_j$ , we obtain:  $\frac{\partial W^{EPTU}}{\partial p_i} - \frac{\partial W^{EPTU}}{\partial p_j} = H(u(c_i) + v(c_j)) - H(v(c_i) + u(c_j))$ . Note that this last expression is always positive when  $c_i > c_j$  (assuming  $u(\cdot) > v(\cdot)$ ). Therefore the property is identical to that derived under the plain utilitarian SWF. A similar result is easily obtained for the prioritarian

<sup>14</sup> This rules out strictly increasing, strictly concave  $g(\cdot)$  functions with  $g(0) = -\infty$ , such as the log function, or  $-(1/x)^\gamma$  with  $\gamma > 0$ .

<sup>15</sup> Let  $F(c) = g'(pu(c))u(c)$ . Then  $\frac{g'(U_i)u(c_i)}{g'(U_j)u(c_j)} > 1$  (resp.  $< 1$ ) for any  $c_i > c_j$  with a zero bequest function reduces to

$F(\cdot) > 0$  (resp.  $< 0$ ) for all  $c$ . But note that  $F'(c) > 0$  for all  $c$  just in case  $-xg''(x)/g'(x) > 1$  for all  $x$ , i.e.,  $g(\cdot)$  has a degree of concavity globally less than unity; that  $F'(c) < 0$  for all  $c$  just in case  $-xg''(x)/g'(x) < 1$  for all  $x$ ; and that  $-xg''(x)/g'(x) = 1$  if  $g(x) = \log x$ .

<sup>16</sup> Recall that our discussion of the social value of risk reduction for the transformed SWFs is limited to the case of independent survival risks.

case, where the property is also identical to the one obtained under the corresponding

$$\text{untransformed benchmark: } \frac{\partial W^{EPTP}}{\partial p_i} > \frac{\partial W^{EPTP}}{\partial p_j} \text{ iff } \frac{g(u(c_i)) - g(v(c_i))}{g(u(c_j)) - g(v(c_j))} > 1.$$

Thus the ex post transformed utilitarian SWF, like plain utilitarianism, is wealth sensitive. The ex post transformed prioritarian SWF is wealth sensitive under the very same conditions (regarding the functional forms of  $g(\cdot)$ ,  $u(\cdot)$ , and  $v(\cdot)$ ) that yield wealth sensitivity for the ex post untransformed prioritarian SWF.<sup>17</sup>

#### IV. Sensitivity to Baseline Risk (and Risk Equity)

It is often argued that policy makers should be sensitive to how risks are distributed in the society. Beginning with Keeney (1980), an extensive theoretical and empirical literature has been devoted to the analysis of social risk equity (see, e.g., Gajdos et al. 2010 for an extensive list of references).

Our model allows us to isolate the effect of individual survival probability on the social value of risk reduction—by considering a case where individual  $i$  has survival probability  $p_i$  in the status quo, individual  $j$  has survival probability  $p_j$ , with  $p_i > p_j$ , and the two individuals have the same wealth. We define (positive) sensitivity to baseline risk as follows:

**Definition 2:** Let  $p_i > p_j$  and  $c_i = c_j$ . A social ranking is (positively) sensitive to baseline risk iff

$$\frac{\partial W}{\partial p_i} < \frac{\partial W}{\partial p_j}.$$

As is well-known,  $VSL_i/VSL_j < 1$ ; hence CBA accords a higher social value to individual risk reduction for individuals at lower survival probability. This is the so-called “dead anyway” effect (Pratt and Zeckhauser 1996). Ex ante prioritarianism also displays the dead-anyway effect:

$$\frac{\partial W^{EAP}}{\partial p_i} / \frac{\partial W^{EAP}}{\partial p_j} \text{ simplifies to } g'(U_i)/g'(U_j) \text{ in the case at hand, which is less than unity because } U_i >$$

$U_j$  and  $g(\cdot)$  is strictly concave, i.e.,  $g'$  is strictly decreasing. By contrast, for plain utilitarianism and ex post untransformed prioritarianism, the social value of risk reduction is insensitive to baseline risk. Note that  $\partial W^U / \partial p_i$  and  $\partial W^{EPU} / \partial p_i$  are, each, solely a function of  $i$ 's wealth; and

thus  $\frac{\partial W / \partial p_i}{\partial W / \partial p_j}$  is, in each case, unity where  $i$  and  $j$  have the same wealth, regardless of their

survival probabilities.

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<sup>17</sup> In particular, with a zero bequest function, the ex post transformed prioritarian SWF—like its untransformed counterpart—is wealth sensitive.

**PROPOSITION II: CBA and ex ante prioritarianism are (positively) sensitive to baseline risk. By contrast, plain utilitarianism and ex post untransformed prioritarianism are insensitive to baseline risk.**

Scholarship on risk reduction often discusses whether a preference for aiding “identified” rather than “statistical” victims is justified. We might say that an individual is an “identified” victim if her probability of surviving the current period, absent governmental intervention, is zero or (more generally) sufficiently low. An immediate implication of Proposition II is that CBA and ex ante prioritarianism, but not plain utilitarianism or ex post untransformed prioritarianism, display a preference for aiding identified victims. Concerns about environmental justice and cumulative risk are also consistent with a social value of risk reduction that is increasing with the individual’s baseline risk, at least to the extent that the baseline risk is determined by environmental exposures.

Sensitivity to baseline risk is closely related to the property of risk equity, examined by Keeney (1980) and Bovens and Fleurbaey (2012)—a preference for equalizing survival probabilities. Imagine that, in the baseline, individual  $j$  has a lower survival probability than individual  $i$ :  $p_j < p_i$ . A policy increases individual  $j$ ’s survival probability to  $p_j + \Delta p$ , and decreases individual  $i$ ’s survival probability to  $p_i - \Delta p$ , leaving  $j$  still at a survival probability no larger than  $i$ . (In other words, the policy secures a Pigou-Dalton transfer in survival probability.) The policy does not change other individuals’ survival probabilities, or anyone’s wealth. Then we say: (1) a policymaking methodology has a *weak* preference for risk equity if it prefers the policy to baseline as long as  $i$  and  $j$  have the same wealth; and (2) a policymaking methodology has a *strong* preference for risk equity if it prefers the policy to baseline regardless of the wealth of the two individuals.

**Definition 3: Let  $p_i' = p_i - \Delta p \geq p_j' = p_j + \Delta p$ , with  $\Delta p > 0$ . Consider a policy  $a$  leading to  $(p_i', p_j')$  and the status quo  $O$  leading to  $(p_i, p_j)$  while leaving unaffected everyone’s wealth and the survival probabilities of everyone excluding  $i$  and  $j$ . A social ranking satisfies a *weak preference for risk equity* iff  $a \succ O$  for  $c_i = c_j$  and a *strong preference for risk equity* iff  $a \succ O$  holds  $\forall c_i, c_j$ .**

Sensitivity to baseline risk is clearly a necessary condition for a weak or strong preference for risk equity. The preference relationship in the definition of weak risk equity preference is satisfied for infinitesimal  $\Delta p$  if and only if the social ranking is positively sensitive to baseline risk. Thus plain utilitarianism and ex post transformed prioritarianism do not satisfy risk equity.

Conversely, we show in the Appendix that CBA<sup>18</sup> and ex ante prioritarianism exhibit risk equity preference in the weak sense. This holds true as long as  $u(\cdot)$  and  $v(\cdot)$  satisfy the standard

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<sup>18</sup> Several qualifications to the result should be noted (see Appendix). First, CBA has been defined here in terms of equivalent variations. CBA in terms of compensating variations does not satisfy risk equity preference. Second, risk

conditions. Moreover, with a logarithmic  $g(\cdot)$  function and a zero bequest function, ex ante prioritarianism satisfies risk equity in the *strong* sense. However, this latter result does not extend beyond this special case (see Appendix).

**PROPOSITION III: CBA and ex ante prioritarianism satisfy risk equity preference in the weak sense. Plain utilitarianism and ex post untransformed prioritarianism do not. Ex ante prioritarianism satisfies risk equity preference in the strong sense under restrictive assumptions regarding  $g(\cdot)$  and individual utility.**

Consider now the ex post transformed utilitarian and prioritarian SWFs (assuming, as above, statistically independent risks). We saw earlier that these SWFs have the very same wealth sensitivity properties as the corresponding benchmark SWFs—regardless of the form of the transformation function  $h(\cdot)$ . This is *not* true for sensitivity to baseline risk/risk equity. We can show that these SWFs are (positively) sensitive to baseline risk, and display a weak preference for risk equity, if  $h(\cdot)$  is convex. They are negatively sensitive to baseline risk if  $h(\cdot)$  is concave (see Appendix).<sup>19</sup> Using Fleurbaey’s (2010) EDE transformation, the ex post transformed utilitarian SWF is not sensitive to baseline risk (because  $h(\cdot)$  is linear) and the ex post transformed prioritarian SWF is sensitive to baseline risk and satisfies a weak preference for risk equity (because  $h(\cdot) = g^{-1}(\cdot)$  is convex).

The following proposition summarizes all of our results thus far regarding the transformed SWFs.

**PROPOSITION IV: The ex post transformed utilitarian SWF is (positively) wealth sensitive; is (positively) sensitive to baseline risk if  $h(\cdot)$  is convex; and displays a weak preference for risk equity if  $h(\cdot)$  is convex. The ex post transformed prioritarian SWF has the same wealth sensitivity properties as the ex post untransformed prioritarian SWF; is (positively) sensitive to baseline risk if  $h(\cdot)$  is convex; and displays a weak preference for risk equity if  $h(\cdot)$  is convex.**

## V. Equal Value of Risk Reduction

Many seem to find equal value of risk reduction—the equal valuation of lives, independent of individual characteristics—to be a desirable feature of a policy-evaluation

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equity preference has been defined here as a preference for Pigou-Dalton transfers *relative to the status quo*. A more general version of weak and strong risk equity preference would change the definition so that  $(p_i, p_j)$  is the result of any policy  $b$  (not necessarily the status quo). CBA even with equivalent variations does *not* satisfy generalized weak risk equity preference.

<sup>19</sup> In the Appendix, we establish that the convexity of  $h(\cdot)$  is a *sufficient* condition for the ex post transformed utilitarian and prioritarian SWFs to be (positively) sensitive to baseline risk, and to display a weak preference for risk equity. However, the convexity of  $h(\cdot)$  may not a *necessary* condition. It may be possible for there to be  $u(\cdot)$ ,  $v(\cdot)$  and  $g(\cdot)$  functions, consistent with the standard utility model and conditions on  $g(\cdot)$ , such that  $h(\cdot)$  is not convex and yet these SWF are positively sensitive to baseline risk and satisfy risk equity. By contrast, in a simpler model, Keeney (1980) finds that a preference for risk equity is equivalent to risk-seeking preferences over the number of fatalities (holding expected fatalities fixed).

methodology (see Baker et al. 2008 and Somanathan 2006). Indeed, this view is reflected in governmental use of population-average rather than differentiated VSL figures. Moreover, Fankhauser et al. (1997) and Johannsson-Stenman (2000) report that one of the most debated issues of the socio-economic chapter of the IPCC Second Assessment Report was the use of a smaller value of life in poor countries than in rich countries.

In this part, we first discuss equal value of risk reduction within the context of the simple model that we set forth in Part II, and that we employed in Parts III and IV to analyze wealth-sensitivity and sensitivity to baseline risk—using CBA, the three benchmark SWFs, and ex post transformed utilitarianism and prioritarianism (with independent survival risks). As shall emerge, equal value of risk reduction is very difficult to achieve within this framework.

We then evaluate a proposal by Baker et al. (2008) that equal value of risk reduction can be achieved via a different kind of  $W$ -function, or by relaxing the standard model of utility.

#### A. Equal Value of Risk Reduction with the Simple Model

Here, we hold fixed the model of Part II—including what we term the standard utility model for VSL, with  $u(c) > v(c)$ ,  $u'(c) > v'(c) \geq 0$  and  $u''(c) \leq 0$ ,  $v''(c) \leq 0$ .

In the model of Part II, individuals are identical except for any differences in their wealth  $c$  or survival probability  $p$ . Thus equal value of risk reduction can be defined as follows.

**Definition 3: A social ranking satisfies equal value of risk reduction iff  $\frac{\partial W}{\partial p_i} = \frac{\partial W}{\partial p_j} \forall p_i, p_j, c_i, c_j$ .**

CBA, plain utilitarianism, ex post transformed utilitarianism, and ex ante prioritarianism clearly *fail* to satisfy equal value of risk reduction. This is because each of these  $W$  functions either has the property of wealth-sensitivity, or the property of sensitivity to baseline risk. Having either of these properties is sufficient—obviously—for *not* satisfying equal value of risk reduction.

By contrast, recall that ex post untransformed prioritarianism,  $W^{EPU}$ , is insensitive to baseline risk. Recall, too, that under *some* conditions regarding  $u(\cdot)$ ,  $v(\cdot)$ , and  $g(\cdot)$ , ex post untransformed prioritarianism is positively or negatively wealth-sensitive (and thus fails equal value of risk reduction). However, there *are* conditions on  $u(\cdot)$ ,  $v(\cdot)$  and  $g(\cdot)$  such that ex post untransformed prioritarianism satisfies equal value of risk reduction.<sup>20</sup>

Recall that ex post *transformed* prioritarianism has the wealth-sensitivity properties of its untransformed counterpart. Thus a necessary condition for  $W^{EPTP}(\cdot)$  to satisfy equal value of risk

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<sup>20</sup> Let  $F(c) = g(u(c)) - g(v(c))$ . Then it is easy to see that ex post untransformed prioritarianism satisfies equal value of risk reduction iff  $F'(c) = 0$ . A sufficient (but not necessary) condition for this to be true is  $v(\cdot) = ku(\cdot)$ , with  $0 < k < 1$  and  $g(x) = \log x$ . Note that ex post untransformed prioritarianism with a zero bequest function exhibits wealth sensitivity (as discussed earlier) and therefore fails to satisfy equal value of risk reduction.

reduction is that  $u(\cdot)$ ,  $v(\cdot)$  and  $g(\cdot)$  fulfill the criteria described in the previous paragraph. By contrast with its untransformed counterpart, however,  $W^{EPTP}(\cdot)$  will be sensitive to baseline risk, positively or negatively, with convex or concave  $h(\cdot)$  functions. Conversely,  $W^{EPTP}(\cdot)$  is insensitive to baseline risk if  $h(\cdot)$  is linear—in which case  $W^{EPTP}(\cdot)$  ranks policies exactly the same way as  $W^{EPUP}$ .

**PROPOSITION V: CBA, plain utilitarianism, ex post transformed utilitarianism, and ex ante prioritarianism do not satisfy equal value of risk reduction. Ex post untransformed prioritarianism satisfies equal value of risk reduction under special conditions regarding  $u(\cdot)$ ,  $v(\cdot)$  and  $g(\cdot)$ . Under those conditions, ex post transformed prioritarianism also satisfies equal value of risk reduction with a linear  $h(\cdot)$  that renders it equivalent to ex post untransformed prioritarianism in the ranking of policies.**

Table I summarizes the wealth- and risk-sensitivity properties of CBA, the three benchmark SWFs, and the two transformed SWFs, and how they fare with respect to equal value of risk reduction.

**Table 1. Summary of properties**

	<i>Positive Wealth Sensitivity</i>	<i>Positive Sensitivity to Baseline Risk</i>	<i>Equal Value of Risk Reduction</i>
CBA	<b>Yes</b>	<b>Yes</b>	<b>No</b>
Plain utilitarian SWF	<b>Yes</b>	<b>No</b>	<b>No</b>
Ex post transformed utilitarian SWF	<b>Yes</b>	<b>Yes</b> if $h(\cdot)$ is convex	<b>No</b>
Ex ante prioritarian SWF	Depends on $g(\cdot)$ , $u(\cdot)$ , and $v(\cdot)$	<b>Yes</b>	<b>No</b>
Ex post untransformed prioritarian SWF	Depends on $g(\cdot)$ , $u(\cdot)$ and $v(\cdot)$ . <b>Yes</b> with a zero bequest function	<b>No</b>	<b>Yes</b> under appropriate restrictions on $g(\cdot)$ , $u(\cdot)$ and $v(\cdot)$
Ex post transformed prioritarian SWF	Depends on $g(\cdot)$ , $u(\cdot)$ and $v(\cdot)$ . <b>Yes</b> with a zero bequest function	<b>Yes</b> if $h(\cdot)$ is convex	<b>Yes</b> under appropriate restrictions on $g(\cdot)$ , $u(\cdot)$ and $v(\cdot)$ and with a linear $h(\cdot)$ function, i.e., the same ranking of policies as $W^{EPUP}$

## B. The Baker et al. proposal

Baker et al. (2008) suggest one may achieve equal value of risk reduction via *weighted* utilitarianism. (In discussing this proposal, for the sake of clarity, we use superscripts to denote

the status quo or alternative policies, so that  $p_i^O$  means  $i$ 's survival probability in the status quo,  $O$ ,  $c_j^a$   $j$ 's wealth with policy  $a$ , and so forth.)

Let  $\beta_i$  be a weighting factor for individual  $i$  equaling  $1 / \left( \frac{\partial U_i}{\partial p_i} \right) \Big|_{U_i^O, p_i^O} = 1 / (u(c_i^O) - v(c_i^O))$ .

Consider a weighted utilitarian SWF for which  $W(a) = \sum_i \beta_i U_i^a$ . This SWF satisfies equal value of risk reduction. If  $i$  has baseline survival probability  $p_i^O$  and baseline wealth  $c_i^O$ , while  $j$  has a possibly different baseline survival probability  $p_j^O$  and possibly different wealth  $c_j^O$ ,  $\frac{\partial W / \partial p_i}{\partial W / \partial p_j}$  in

the baseline is just  $\frac{\beta_i (u(c_i^O) - v(c_i^O))}{\beta_j (u(c_j^O) - v(c_j^O))} = \frac{1 / (u(c_i^O) - v(c_i^O)) (u(c_i^O) - v(c_i^O))}{1 / (u(c_j^O) - v(c_j^O)) (u(c_j^O) - v(c_j^O))} = 1$ .

However, closer inspection suggests that this SWF is problematic. The most natural interpretation of the Baker et al. (2008) proposal is that the weights are assigned to each individual depending upon her baseline characteristics in  $O$ , but are then held “rigid”: in order to calculate the sum of weighted utilities for any policy  $a$ , the weighting factor for individual  $i$  is  $\beta_i$ , regardless of  $i$ 's characteristics (wealth and survival probability) in  $a$ . This approach violates the “anonymity” or “impartiality” axiom – a basic principle that any minimally plausible SWF should satisfy. Assume that, in policy  $a$ , individuals have wealth and survival probabilities  $((c_1, p_1), (c_2, p_2), \dots (c_N, p_N))$ , while in policy  $b$  these pairs are permuted. Then “anonymity”/“impartiality” requires that a SWF be indifferent between  $a$  and  $b$ ; but the form of weighted utilitarianism now under discussion need not be.<sup>21</sup>

A different interpretation is the weights are not “rigid,” but instead assigned by a weighting function. In other words,  $W(a) = \sum_i \beta(p_i^a, c_i^a) U_i^a$ , where  $\beta(p_i^a, c_i^a) = 1 / [u(c_i^a) - v(c_i^a)]$ . This SWF can violate the Pareto principle (at least if the bequest function is zero). Consider a policy that departs from baseline by increasing some individuals' wealth, without changing anyone's survival probability. Then the Pareto principle obviously requires that the policy be

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<sup>21</sup> It might be protested that failures of anonymity require “large” rather than small departures from the baseline—and Baker et al. (2008) are only proposing their SWF for small changes—but this is not true. Imagine that, in the baseline, one individual has wealth  $c$  and another wealth  $c^*$ , which is slightly larger, and that they have the same survival probability. Imagine that the policy increases the first individual's wealth to  $c^*$  and decreases the second's to  $c$ . Then anonymity requires that this “small” departure from the baseline be ranked equally good as baseline; but the “rigid” form of weighted utilitarianism will not do that.

A referee observed that CBA also violates anonymity. Assume that two individuals have identical status quo wealth but different survival probabilities. If a policy swaps these probabilities (and changes nothing else), anonymity requires social indifference between the status quo and the policy; but CBA will generally *not* be indifferent, since the status quo expected marginal utilities of wealth used to calculate the two individuals' equivalent variations will be different. CBA's violation of anonymity may be seen as a reason to prefer an SWF that satisfies anonymity (such as  $W^U$ ,  $W^{EPUP}$ ,  $W^{EAP}$ ,  $W^{EPTP}$ , or  $W^{EPTU}$ ) rather than CBA or weighted utilitarianism.

preferred, but the SWF now being discussed will be indifferent between policy and baseline (when the bequest function is zero).<sup>22</sup>

Although Baker et al. (2008) focus on the weighted-utilitarian SWF, they suggest in a footnote that equal value of risk reduction might also be achieved in an alternative manner—by relaxing the standard utility model. We find this suggestion more plausible. Consider, in particular, the possibility of setting  $u(c) = v(c) + k$ ,  $k > 0$ ,  $u'(c) = v'(c) > 0$ ,  $u''(c) = v''(c) \leq 0$ . It should be stressed that these assumptions are perfectly consistent with expected utility theory. Nor do they seem absurd. If  $c$  is defined as wealth *after* insurance premiums and payouts,  $u'(\cdot)$  and  $v'(\cdot)$  might plausibly be equal, since optimal insurance equalizes the marginal utility of money across states of the world.

With these specifications of  $u(\cdot)$  and  $v(\cdot)$ , plain utilitarianism will satisfy equal value of risk reduction.<sup>23</sup> However,  $W^{CBA}$ ,  $W^{EAP}$ , and  $W^{EPUP}$  continue to violate equal value of risk reduction.<sup>24</sup>

## VI. Catastrophe Aversion

Slovic et al. (1984) asked: “How should a single accident that takes  $N$  lives be weighted relative to  $N$  accidents, each of which takes a single life?”. As an answer to this question, it is often advanced that, for a given number of expected fatalities, big accidents are worse. This catastrophe aversion preference is included in the practice of several governmental agencies

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<sup>22</sup> Admittedly,  $W^{EPUP}$ ,  $W^{EPTU}$ , and  $W^{EPTP}$  can also violate the Pareto principle. However, such violation only occurs when the social planner is choosing under conditions of uncertainty. By contrast, the weighted-utilitarian SWF under discussion in this paragraph can violate the Pareto principle even if the planner knows, for certain, how individuals will be affected. (Even if  $p_i^a$  is one or zero for all individuals and actions, a violation of the Pareto principle can occur.) Arguably, an SWF which conflicts with the Pareto principle under conditions of certainty is *especially* problematic. See generally Adler (2012), Chapter 7.

<sup>23</sup> Note that  $\frac{\partial W^U}{\partial p_i} = u(c_i) - v(c_i) = k$  for all  $c_i, p_i$ .

<sup>24</sup> Assume that for all  $c$ ,  $u(c) - v(c) = k > 0$ , and thus  $u'(c) = v'(c)$ . Consider two individuals with survival

probabilities  $p_i$  and  $p_j$  and wealth  $c_i$  and  $c_j$ . Then  $\frac{VSL_i}{VSL_j} = \frac{u'(c_j)}{u'(c_i)}$ , which is not unity with  $c_i \neq c_j$  if  $u(\cdot)$  is strictly

concave (rather than linear).  $\frac{\partial W^{EPUP} / \partial p_i}{\partial W^{EPUP} / \partial p_j} = \frac{g(u(c_i)) - g(u(c_i) - k)}{g(u(c_j)) - g(u(c_j) - k)}$ , which is not unity with  $c_i \neq c_j$ , since  $g(\cdot)$  is

strictly concave. Finally,  $\frac{\partial W^{EAP} / \partial p_i}{\partial W^{EAP} / \partial p_j} = \frac{g'(u(c_i) - (1 - p_i)k)}{g'(u(c_j) - (1 - p_j)k)}$ , which is not unity with  $p_i = p_j$  and  $c_i \neq c_j$  given the

strict concavity of  $g(\cdot)$ .

We do not establish results for  $W^{EPTU}$  and  $W^{EPTP}$  given the utility model  $u(c) = v(c) + k$ ,  $k > 0$ ,  $u'(c) = v'(c) > 0$ ,  $u''(c) = v''(c) \leq 0$ . Obviously, with a linear  $h(\cdot)$  function, the results are the same as for  $W^U$  and  $W^{EPUP}$ , respectively. Matters become more complex if  $h(\cdot)$  is allowed to be non-linear and, indeed, perhaps neither concave nor convex.

(Bedford 2013), although the public does not seem to display such a preference (Jones-Lee and Loomes 1995; Covey et al. 2010; Rheinberger 2010).

Keeney (1980) offers a formal definition of catastrophe aversion. Assume that policy  $a$  has a probability  $\pi_d$  of  $d$  premature deaths and a probability  $(1 - \pi_d)$  of no deaths, while policy  $b$  has a probability  $\pi_{d'}$  of  $d'$  premature deaths and a probability  $(1 - \pi_{d'})$  of no deaths. Assume, further, that the two policies have the same number of expected deaths ( $d\pi_d = d'\pi_{d'}$ ), but  $d$  is smaller than  $d'$ . Then a policymaking tool is catastrophe-averse in Keeney's sense (for short, "Keeney catastrophe averse") if it prefers policy  $a$  to  $b$ . As noted by Keeney (1980), catastrophe aversion implies a preference for a policy in which a few ( $d$ ) individuals die for sure to an alternative in many ( $N$ ) die together with probability  $d/N$  (and survive with probability  $1 - d/N$ ).

The concept of a mean-preserving spread (Rothschild and Stiglitz 1970) suggests a natural generalization of Keeney catastrophe aversion. Let  $D$  be a random variable representing the number of fatalities. Let us say that a policymaking tool is "globally catastrophe averse" if it dislikes a mean-preserving spread of  $D$ . Note that Keeney catastrophe aversion is a particular case of global catastrophe aversion in which  $D$  is binary with one outcome having zero fatalities.

**Definition 5: Let  $D'$  be a mean-preserving spread of  $D$ , both random variables. Consider a policy  $a$  leading to  $D$  fatalities and a policy  $b$  leading to  $D'$  fatalities. A social ranking exhibits strong global catastrophe aversion iff  $a \succ b$  and weak global catastrophe aversion iff  $a \succ b$  holds whenever all individuals have equal wealth.**

In order to decide whether a  $W$ -function satisfies catastrophe aversion, we need to be able to associate each policy with a probability distribution  $D$  over fatalities. In the initial statement of our simple model (Part II), each policy was characterized as an array of individual wealth amounts, plus a state-dependent assignment of each individual to the status "dead" or "alive." States have exogenous probabilities. Such a characterization of a given policy  $a$  determines the probability distribution  $D$  over fatalities associated with  $a$ .

However, in our subsequent analysis, we have generally simplified the description of policies—so that each is characterized *just* as a vector of individual survival probabilities, plus wealth amounts. In order to determine the social value of risk reduction  $\partial W / \partial p_i$  for a given  $W$ -function—and in particular to assess whether  $\partial W / \partial p_i$  has the properties of wealth-sensitivity and sensitivity to baseline risk—it often *suffices* to know how the  $W$ -function ranks policies characterized in this simpler way.<sup>25</sup>

For purposes of discussing the property of catastrophe aversion, we must revert to thinking of policies in the initial, fuller, manner: as state-dependent assignments of individuals to

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<sup>25</sup> Recall also that such ranking was generally well defined for CBA and the three benchmarks, but not for the two transformed SWFs absent additional information about risk correlation. Thus, in discussing the social value of risk reduction for the transformed SWFs, we have assumed statistically independent risks.

the status “alive” or “dead,” plus individual wealth amounts. Why? Merely knowing the vector of individual probabilities associated with a given policy  $a$  does not determine the distribution  $D$  over fatalities with which  $a$  is associated.

It is clear that CBA and the three benchmark SWFs violate weak and hence strong global catastrophe aversion. Why? Consistent with Slovic et al.’s (1984) question, assume for instance that there are  $N$  states of the world and  $N$  individuals. With policy  $a$ , exactly one person dies in each state. With policy  $b$ , all  $N$  die in one state and survive in every other. Catastrophe aversion preference requires that policy  $a$  be preferred. But in this example, both CBA and the three benchmark SWFs would be indifferent between the two policies.

An interesting topic, one we do not pursue at length, is to explore the catastrophe-aversion or proneness properties of CBA and the three benchmarks *given* various constraints on the correlation of individual risks. If policy  $a$  is less catastrophic than  $b$ , *and* survival outcomes in each policy are correlated in a certain manner, then it *might* be the case that CBA or one of the three benchmarks prefers  $a$  to  $b$ , or  $b$  to  $a$ .

This observation relates to Keeney (1980) and to important subsequent work by Bovens and Fleurbaey (2012). These scholars assume independent survival risks, and under that constraint show a link between catastrophe-proneness and a preference for equalizing individual risks. Translating these results into our framework, consider the following. Let  $O$  be the status quo, and  $b$  an alternative policy, such that (1) survival risks are statistically independent with both policies and (2)  $b$  is more catastrophic in Keeney’s sense, i.e., Keeney catastrophe aversion requires a preference for  $O$ . Then it can be demonstrated that  $b$  can be reached from  $O$  via a series of Pigou-Dalton transfers of individual survival probabilities. If we assume that individuals have equal wealth, CBA and ex ante prioritarianism will prefer  $b$  to  $O$ —because these  $W$ -functions satisfy risk equity preference with equal wealth. In short, given statistically independent survival risks, CBA and ex ante prioritarianism are weakly Keeney catastrophe-prone. (By contrast, in the case considered two paragraphs above, without independent risks, CBA and ex ante prioritarianism are neutral between  $a$  and  $b$ .)<sup>26</sup>

In any event, the basic and straightforward result here is that—without further constraints on the correlation of individual survival risks—CBA and the three benchmarks fail weak and strong catastrophe aversion as defined in Definition 5.

By contrast, a striking fact is that ex post *transformed* utilitarianism and ex post *transformed* prioritarianism will *satisfy* weak<sup>27</sup> Keeney catastrophe aversion if the social transformation function  $h(\cdot)$  is strictly concave. To see this, consider a population of  $N$  individuals out of which  $d$  individuals will die if a catastrophe occurs. All have the same wealth

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<sup>26</sup> Other illustrative examples relating (ex ante) distributions of individual probabilities and (ex post) distributions of fatalities are discussed in Gajdos et al. (2010).

<sup>27</sup> Meaning that individuals have equal wealth.

$c$ ; let “ $u$ ” and “ $v$ ” denote  $u(c)$  and  $v(c)$ , respectively. Keep the expected number of deaths  $n$  constant, so that the probability of catastrophe is  $\pi = n/d$ .

Consider ex post transformed utilitarianism. If  $N-d$  individuals are alive, the social value of that state, according to ex post transformed utilitarianism, is  $h((N-d)u+dv)$ . Accordingly, social welfare is equal to  $W(d) = (n/d)h(Nu+d(v-u)) + (1-(n/d))h(Nu)$ . Weak Keeney catastrophe aversion means that social welfare must be decreasing in the number of fatalities  $d$ . That is, there is weak Keeney catastrophe aversion if  $W'(d) < 0$ . We easily obtain  $W'(d) = -(n/d^2)[h(Nu+d(v-u))-h(Nu)] + (n/d)(v-u)h'[Nu+d(v-u)]$ . It is straightforward then that  $W'(d)$  is negative for all parameters  $N, u, v$  and  $d$  if  $(h(s)-h(r))/(s-r) < h'(r)$  for all  $s$  and  $r$  such that  $s > r$ , which indeed holds if  $h(\cdot)$  is strictly concave.<sup>28</sup>

It is easy to generalize this result to weak global catastrophe aversion. If the random number of fatalities is  $D$ , social welfare under ex post transformed utilitarianism simply becomes  $Eh((N-D)u+Dv)$  in which  $E$  is the expectation operator over  $D$ . It is immediate then that there is weak global catastrophe aversion if  $h((N-d)u+dv)$  is strictly concave in  $d$ , that is, if  $h$  is strictly concave.

A parallel analysis shows that ex post transformed prioritarianism satisfies weak Keeney and global catastrophe aversion if the transformation function is concave. Continuing the discussion of the previous paragraph: social welfare under ex post transformed *prioritarianism* simply becomes  $Eh((N-d)g(u)+dg(v))$ . There is weak global catastrophe aversion if  $h((N-d)g(u)+dg(v))$  is concave in  $d$ , that is, if  $h$  is concave.<sup>29</sup>

**PROPOSITION VI: Ex post transformed utilitarianism and prioritarianism satisfy weak Keeney and global catastrophe aversion if the transformation function  $h(\cdot)$  is strictly concave. CBA, plain utilitarianism, ex post untransformed prioritarianism, and ex ante prioritarianism fail to satisfy Keeney and global catastrophe aversion.**

It is also worth noting that Fleurbaey’s (2010) EDE transformation function  $h^{EDE}(\cdot)$ , combined with utilitarianism or prioritarianism, fails Keeney and global catastrophe aversion.<sup>30</sup>

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<sup>28</sup> Indeed,  $W'(d)$  is negative for all parameters  $N, u, v$ , and  $d$  if and only if  $h(\cdot)$  is strictly concave. However, it may be possible for there to be  $u(\cdot)$  and  $v(\cdot)$  functions, consistent with the standard utility model, such that  $W'(d)$  is negative even with a non-concave  $h(\cdot)$ . We therefore state all the results concerning catastrophe aversion in terms of sufficient conditions rather than equivalences.

<sup>29</sup> Although ex post transformed utilitarianism and prioritarianism satisfy weak catastrophe aversion with an appropriate transformation function, they are not necessarily catastrophe averse when individuals can vary in their wealth.

<sup>30</sup> Fleurbaey (2010) informally discusses Keeney catastrophe aversion, and suggests that it may make more sense to reduce an independent risk than a risk that hits everyone equally. The intuition is that, when the number of expected fatalities is given, one may prefer a catastrophe with a higher number of fatalities since this reduces ex post inequality. At the limit, if everyone will be either alive or dead, there is maximal ex post equality. This also relates to the idea that “misery loves company” (Bovens and Fleurbaey 2012).

As discussed in Part I, if the underlying SWF is utilitarian,  $h^{EDE}(\cdot)$  is linear; if the underlying SWF is prioritarian,  $h^{EDE}(\cdot)$  is strictly *convex*.<sup>31</sup>

## V. Conclusion

Cost-benefit analysis (CBA) evaluates the social gain from reductions in mortality risk using the concept of the value per statistical life (VSL). As a guide to public policy, CBA using VSL exhibits several properties concerning the social value of reducing mortality risk to different people that some commentators perceive to be undesirable, such as positive sensitivity to wealth and unequal value of risk reduction.

We evaluate different versions of a utilitarian or prioritarian social welfare function (SWF), and find that these do not necessarily share the same properties as CBA. CBA exhibits positive wealth sensitivity and positive sensitivity to baseline risk (the dead-anyway effect). The utilitarian SWFs (plain and ex post transformed) also exhibit positive wealth sensitivity, but the prioritarian SWFs (ex ante, ex post untransformed, and ex post transformed) may or may not do so, depending on parameter assumptions. The ex ante prioritarian SWF exhibits positive sensitivity to baseline risk, but the plain utilitarian SWF and ex post untransformed prioritarian SWF are neutral to baseline risk; and the ex post transformed utilitarian and prioritarian SWFs are positively sensitive to baseline risk if the transformation function is convex but *negatively* sensitive if this function is concave.

Further, all of these methodologies satisfy a property of risk equity preference<sup>32</sup> if and only if they are positively sensitive to baseline risk. None of the approaches value risk reductions equally in a population, except for the ex post prioritarian SWFs under restrictive conditions. CBA does not exhibit catastrophe aversion, and in general neither do the SWFs, although the ex post transformed utilitarian and prioritarian SWFs will do so with a concave transformation function.

It is also instructive to note that the trio of properties characteristic of CBA—positive wealth sensitivity, positive sensitivity to baseline risk/risk equity preference, no catastrophe aversion—is not inherent to any of the SWFs we have considered, although it can be replicated by ex ante prioritarianism with appropriate parametric assumptions.

We conclude with three possible research directions that are motivated by limitations of the current work.

First, we have studied how a policy making assessment method (CBA or the benchmark SWFs) would prioritize risk reductions in society, and how this depends on the properties of the

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<sup>31</sup> As noted in Part I, where  $w = \sum_{i=1}^N g(u_i)$ ,  $h^{EDE}(w) = g^{-1}(w/N)$ . With  $g(\cdot)$  strictly concave,  $g^{-1}(\cdot)$  is strictly convex.

<sup>32</sup> Strictly, “weak” risk equity preference, where the individuals involved have equal wealth.

SWF and utility functions. But, importantly, we have not studied how those methods would allocate the financial costs of the risk-reduction program across individuals, or how the total social value of reducing risk varies with the assessment method and allocation of costs. It may be useful in the future to study the general problem, namely how the different settings would fare simultaneously with financial and risk distributional effects. This would generalize previous models on public provision of safety (Jones-Lee 1989; Pratt and Zeckhauser 1996), assuming some specific tax structures and SWFs.

Second, we have indicated in Part II that under transformed settings (namely ex post transformed utilitarian and ex post transformed prioritarian SWFs), it is not possible in general to express the social value of risk reduction as a function of individuals' survival probabilities, without also specifying the correlation across individual mortality risks. Following Keeney (1980) and others, we have often assumed that individual mortality risks are statistically independent. This suggests that there is a need to generalize this analysis to different assumptions about dependence among individual risks. See Bommier and Zuber (2008) and Bovens and Fleurbaey (2012, section 7) for early analyses.

Finally, we remind the reader that we have analyzed only a few SWFs. In particular we have not studied how the rank-weighted and leximin SWFs would evaluate the social value of risk reduction; that is also an important topic for future research.

## Appendix

We provide here proofs of the claims relating to sensitivity to baseline risk/risk equity for CBA and the SWFs that were made but not proven in the main text.

### 1. CBA and Risk Equity

#### (a) CBA with Equivalent Variations Satisfies a Weak Preference for Risk Equity

Individual  $i$  has survival probability  $p_i$ , individual  $j$  has survival probability  $p_j$ , with  $p_j < p_i$ . Both individuals have the same wealth  $c$ . If a policy decreases  $i$ 's survival probability by  $\Delta p$  and increases  $j$ 's by the same amount, then the individuals' equivalent variations for the policy are as follows, with  $\Delta c_i < 0$  and  $\Delta c_j > 0$ :

$$(1) \quad u(c + \Delta c_i)p_i + v(c + \Delta c_i)(1 - p_i) = u(c)(p_i - \Delta p) + v(c)(1 - p_i + \Delta p)$$

$$(2) \quad u(c + \Delta c_j)p_j + v(c + \Delta c_j)(1 - p_j) = u(c)(p_j + \Delta p) + v(c)(1 - p_j - \Delta p).$$

Equation (1) simplifies to:

$$(3) \quad [u(c) - u(c + \Delta c_i)]p_i + [v(c) - v(c + \Delta c_i)](1 - p_i) = [u(c) - v(c)] \Delta p.$$

Similarly, (2) simplifies to:

$$(4) \quad [u(c + \Delta c_j) - u(c)]p_j + [v(c + \Delta c_j) - v(c)](1 - p_j) = [u(c) - v(c)] \Delta p.$$

Thus:

$$(5) \quad [u(c) - u(c + \Delta c_i)]p_i + [v(c) - v(c + \Delta c_i)](1 - p_i) = \\ [u(c + \Delta c_j) - u(c)]p_j + [v(c + \Delta c_j) - v(c)](1 - p_j).$$

Use the abbreviations  $A^*$  to mean  $[u(c) - u(c + \Delta c_i)]$ ,  $B^*$  to mean  $[v(c) - v(c + \Delta c_i)]$ ,  $A$  to mean  $[u(c + \Delta c_j) - u(c)]$  and  $B$  to mean  $[v(c + \Delta c_j) - v(c)]$ .

Because  $u' > v'$ ,  $A^* > B^*$  and therefore  $p_i A^* + (1 - p_i) B^* > p_j A^* + (1 - p_j) B^*$ .

It is therefore impossible that  $-\Delta c_i = \Delta c_j$ . If that were the case, we would have a contradiction. It would follow (given the weak concavity of  $u(\cdot)$  and  $v(\cdot)$ ) that  $A^* \geq A$  and  $B^* \geq B$ , and thus that  $p_i A^* + (1 - p_i) B^* > p_j A + (1 - p_j) B$ , i.e., the left side of equation 5 would be greater than the right. Note, finally, that the term  $p_i A^* + (1 - p_i) B^*$ , the left side of equation 5, is *decreasing* in  $\Delta c_i$ . (This can be seen by differentiating that term with respect to  $\Delta c_i$ .) Thus, for equation (5) to hold, it must be that  $-\Delta c_i < \Delta c_j$ , or the sum of equivalent variations is positive and risk equity preference holds.

#### (b). CBA with Equivalent Variations Satisfies Risk Equity Preference only for Pigou-Dalton Transfers relative to the status quo, and not in general

Let  $v(c) = \ln c$  and  $u(c) = 2v(c)$ , with  $c > 1$ . Assume that all individuals have income 100, and that in the status quo both individuals  $i$  and  $j$  have survival probability 0.3. Let policy  $a$  be such their survival probabilities are, respectively, 0.1 and 0.9; while policy  $b$  is such that their survival probabilities are 0.4 and 0.6. Everyone else has the same survival probabilities in policy  $a$  and  $b$ . Then policy  $b$  is an equalizing transfer relative to policy  $a$ . However it can be verified that individual  $i$  and  $j$  have equivalent variations for policy  $a$  of, respectively, -51 and 738; while their equivalent variations for policy  $b$  are 43 and 189. Thus the sum of equivalent variations prefers policy  $a$ .

(c). CBA with Compensating Variations Can Violate a Weak Preference for Risk Equity

As before, let individual  $i$  have survival probability  $p_i$ , and individual  $j$  survival probability  $p_j$ , with  $p_j < p_i$ . Both individuals have the same wealth  $c$ . If a policy decreases  $i$ 's survival probability by  $\Delta p$  and increases  $j$ 's by the same amount, then the individuals' *compensating* variations for the policy are as follows, with  $\Delta c_i < 0$  and  $\Delta c_j > 0$ :

$$(1^*) \quad u(c)p_i + v(c)(1 - p_i) = u(c - \Delta c_i)(p_i - \Delta p) + v(c - \Delta c_i)(1 - p_i + \Delta p)$$

$$(2^*) \quad u(c)p_j + v(c)(1 - p_j) = u(c - \Delta c_j)(p_j + \Delta p) + v(c - \Delta c_j)(1 - p_j - \Delta p).$$

To see a simple case where  $-\Delta c_i > \Delta c_j$  and thus weak risk equity preference fails, let  $v(\cdot) = 0$ ,  $p_i = 1$ , and  $p_j = 0$ , and  $u(\cdot)$  be the square root function. Equation (1\*) simplifies to:

$$(3^*) \quad c \frac{1 - (1 - \Delta p)^2}{(1 - \Delta p)^2} = -\Delta c_i.$$

Equation (2\*) simplifies to  $\Delta c_j = c$ . A little manipulation of (3\*) shows that, if  $\Delta p > 1 - \sqrt{1/2} \approx 0.3$ , then  $-\Delta c_i > c$ .

2. *Ex Ante Prioritarianism and Risk Equity*

We stated in Part IV that ex ante prioritarianism satisfies a weak preference for risk equity. This can be easily demonstrated. Assume, as before,  $p_i > p_j$  and both individuals have the same wealth  $c$ . Assume policy  $a$  decreases  $i$ 's survival probability by  $\Delta p$  and increases  $j$ 's by  $\Delta p$ , where  $p_j + \Delta p \leq p_i - \Delta p$ . Let  $U_i^a$  denote  $i$ 's expected utility for the policy, i.e.,  $(p_i - \Delta p)u(c) + (1 - p_i + \Delta p)v(c)$ . Similarly,  $U_j^a = (p_j + \Delta p)u(c) + (1 - p_j - \Delta p)v(c)$ . According to ex ante prioritarianism, the change in social value associated with the policy is  $g(U_i^a) + g(U_j^a) - g(U_i) - g(U_j)$ , so the policy is preferred iff  $g(U_j^a) - g(U_j) > g(U_i) - g(U_i^a)$ . Note, now, that  $U_j^a - U_j = U_i - U_i^a = \Delta p [u(c) - v(c)]$ , which is greater than zero because  $u(c) > v(c)$ . Moreover, because  $u(c) > v(c)$  and  $p_j + \Delta p \leq p_i - \Delta p$ , it follows that  $U_j^a \leq U_i^a$ . Thus, by strict concavity of  $g(\cdot)$ ,  $g(U_j^a) - g(U_j) > g(U_i) - g(U_i^a)$ .

In Part IV, we also indicated that ex ante prioritarianism with a logarithmic  $g(\cdot)$  function and a zero bequest function satisfies a strong preference for risk equity (i.e., even where the

individuals do not have the same wealth). Indeed, we then have  $g(U_j^a) - g(U_j) - g(U_i) + g(U_i^a) = \log(p_j + \Delta p) - \log p_j + \log(p_i - \Delta p) - \log p_i$ , which is always positive as long as  $p_j + \Delta p \leq p_i - \Delta p$ . Nevertheless the result that ex ante prioritarianism satisfies risk equity in the strong sense does not extend beyond the special logarithmic case. Indeed, with a zero bequest function, the logarithmic function is the only strictly concave  $g(\cdot)$  function with this property. To see that, observe that wealth has no effect on  $g(U_j^a) - g(U_j)$  for an infinitesimal  $\Delta p$  only when  $F(c) = g'(pu(c))u(c)$  is independent of  $c$ . We obtain  $F'(c) = g''(pu(c))pu'(c)u(c) + g'(pu(c))u'(c)$ , so that  $F'(c) = 0$  for all  $c$  is equivalent to  $-xg''(x)/g'(x) = 1$  for all  $x$ , or  $g(\cdot) = \log$ .

### 3. Ex Post Transformed Utilitarian and Prioritarian SWFs

Assume  $c_i = c_j = c$ , and denote  $u(c) = u$  and  $v(c) = v$ . Using again the equality derived in Part II, we easily obtain  $\frac{\partial W^{EPTU}}{\partial p_i} - \frac{\partial W^{EPTU}}{\partial p_j} = 2(p_i - p_j)[H(u+v) - \frac{1}{2}(H(2u) + H(2v))]$ . This expression is always negative when  $p_i > p_j$  for all  $u$  and  $v$  iff  $H(x)$ , and thus also iff  $h(x)$ , is convex. A parallel demonstration can be developed for the case of ex post transformed prioritarianism.

It is not difficult to show that ex post transformed utilitarianism also displays risk equity under the same condition. Assume as before  $c_i = c_j = c$ . The demonstration is similar to Keeney (1980)'s. Let us define  $p_i = p + \Delta$  and  $p_j = p - \Delta$  so that

$$\begin{aligned} W^{EPTU} &\equiv W(\Delta) = (p - \Delta^2)H(2u) + (2p(1-p) + 2\Delta^2)H(u+v) + ((1-p)^2 - \Delta^2)H(2v) \\ &= W(0) + 2\Delta^2[H(u+v) - \frac{1}{2}(H(2u) + H(2v))]. \end{aligned}$$

Therefore, for all  $u$  and  $v$ ,  $W(\Delta)$  is decreasing in  $\Delta$ , iff  $H(x)$ , and thus also iff  $h(x)$ , is convex. Again, an exact parallel demonstration can be obtained for the ex post transformed prioritarian case.

Note that the results in these paragraphs establish that a convex  $h(\cdot)$  is sufficient to yield sensitivity to baseline risk, and risk equity, for any  $u(\cdot)$  and  $v(\cdot)$ . They do not establish that, for some particular  $u(\cdot)$  and  $v(\cdot)$ , a convex  $h(\cdot)$  is necessary for sensitivity to baseline risk and risk equity. Proposition IV in the text concerning the transformed SWFs and sensitivity to baseline risk/risk equity is therefore formulated with the convexity of  $h(\cdot)$  as a sufficient condition.

## References

- Adler, M., 2012. *Well-Being and Fair Distribution: Beyond Cost-Benefit Analysis*. Oxford University Press.
- Baker, R., Chilton S., Jones-Lee M. and H. Metcalf, 2008. Valuing lives equally: Defensible premise or unwarranted promise?, *Journal of Risk and Uncertainty* 36, 125-38.
- Bedford T., 2013, Decision making for group risk reduction: Dealing with epistemic uncertainty, *Risk Analysis*, DOI: 10.1111/risa.12047
- Ben-Porath, E., Gilboa I., and D. Schmeidler, 1997. On the measurement of inequality under uncertainty, *Journal of Economic Theory* 75, 194–204.
- Blackorby, C., Bossert W., and D. Donaldson, 2005. *Population Issues in Social Choice Theory, Welfare Economics and Ethics*. Cambridge: Cambridge University Press.
- Blackorby, C., and D. Donaldson. 1986. Can risk-benefit analysis provide consistent policy evaluations of projects involving loss of life? *Economic Journal* 96, 758-773.
- Bommier A. and S. Zuber, 2008, Can preferences for catastrophe avoidance reconcile social discounting with intergenerational equity?, *Social Choice and Welfare* 31, 415-34.
- Bossert, W. and J.A. Weymark, 2004. Utility in social choice. In S. Barbera, P.J. Hammond and C. Seidl, eds, *Handbook of Utility Theory*, vol. 2, 1099-1177. Boston: Kluwer Academic.
- Bovens, L. and M. Fleurbaey, 2012. Evaluating life and deaths prospects, *Economics and Philosophy* 28, 217-49.
- Covey, J., A. Robinson, M. Jones-Lee, and G. Loomes, 2010. Responsibility, scale and the valuation of rail safety, *Journal of Risk and Uncertainty* 40, 85-108.
- Drèze, J. and N. Stern, 1987. The theory of cost-benefit analysis. In A. Auerbach and M. Feldstein, eds., *Handbook of Public Economics*, vol. 2, 909-989. Amsterdam: Elsevier Science.
- European Commission, DG Environment, 2001. Recommended interim values for the value of preventing a fatality in DG Environment cost benefit analysis. See at: [http://ec.europa.eu/environment/enveco/others/pdf/recommended\\_interim\\_values.pdf](http://ec.europa.eu/environment/enveco/others/pdf/recommended_interim_values.pdf)
- Fankhauser, S., R. S. J. Tol and D.W. Pearce, 1997. The aggregation of climate change damages: A welfare theoretic approach, *Environmental and Resource Economics* 10, 249–266.
- Fleurbaey, M., 2010. Assessing risky social situations, *Journal of Political Economy* 118, 649-680.
- Gajdos, T., Weymark J.A. and C. Zoli, 2010, Shared destinies and the measurement of social risk equity, *Annals of Operations Research* 176, 409-24.
- Hammitt, J.K. and N. Treich, 2007. Statistical vs. identified lives in benefit-cost analysis, *Journal of Risk and Uncertainty* 35, 45-66.
- HM Treasury, 2011. *The Green Book: Appraisal and Evaluation in Central Government*, London.

- Institute of Medicine, 2006. *Valuing Health for Regulatory Cost-Effectiveness Analysis*, National Academies Press, Washington, D.C.
- Johansson-Stenman, O., 2000. On the value of life in rich and poor countries and distributional weights beyond utilitarianism, *Environmental and Resource Economics* 17, 299-310.
- Jones-Lee, M.W., 1989, *The Economics of Safety and Physical Risk*, Oxford, Basic Blackwell.
- Jones-Lee, M.W. and G. Loomes, 1995, Scale and context effects in the valuation of transport safety, *Journal of Risk and Uncertainty* 11, 183-203.
- Jonsen, A. R., 1986. Bentham in a box: technology assessment and health care allocation, *Law, Medicine and Health Care* 14, 172-4.
- Kaplow, L., 2008. *The Theory of Taxation and Public Economics*, Princeton University Press.
- Keeney, R.L., 1980. Equity and public risk, *Operations Research* 28, 527-34.
- Lazarus, R.J., 1993. Pursuing “Environmental Justice”: The distributional effects of environmental protection, *Northwestern University Law Review* 87, 787-856.
- Neumann, P.J., and M.C. Weinstein, 2010. Legislating against use of cost-effectiveness information, *New England Journal of Medicine* 363, 1495-1497.
- Pratt, J. W., and R. J. Zeckhauser, 1996. Willingness to pay and the distribution of risk and wealth, *Journal of Political Economy* 104, 747-63.
- Rheinberger, C.M., 2010, Experimental evidence against the paradigm of mortality risk aversion, *Risk Analysis* 30, 590-604.
- Robinson, L.A., 2007. How US government agencies value mortality risk reductions. *Review of Environmental Economics and Policy* 1, 283-299.
- Rothschild, M. J. and J. Stiglitz, 1970. Increasing risk I: A definition, *Journal of Economic Theory* 2, 225-43.
- Slovic, P., 2000. *The Perception of Risk*. Earthscan Publisher.
- Slovic P., Lichtenstein S. and B. Fischhoff, 1984, Modeling the societal impact of fatal accidents, *Management Science* 30, 464-74.
- Somanathan, E., 2006. Valuing lives equally: Distributional weights for welfare analysis, *Economics Letters* 90, 122-25.
- Ulph, A., 1982. The role of ex ante and ex post decisions in the valuation of life, *Journal of Public Economics* 18, 265-76.
- Viscusi, W.K., 2009. The devaluation of life, *Regulation and Governance* 3, 103-127.